Eastern Multi-Academy Trust
Empower - Motivate - Aspire - Transform

Upwell Academy

Calculations Policy

Aims:

- To provide a concrete, pictorial and abstract (CPA) teaching approach that enables us to deliver on our mathematics intent statement, which also develops our children's conceptual understanding
- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations

How to use this policy:

- Use the policy as the basis of your planning but ensure you use previous or following stages' guidance to allow for personalised learning
- Cross reference with the National Curriculum end of year number skills expectations for each year group
- Use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Always model a CPA approach to allow children to see the links and to make rich connections
- Teach addition and subtraction at the same time to demonstrate how they link and the inverse law
- Teach multiplication and division at the same time to demonstrate the links between these operations and the inverse law
- All written methods should be presented to the children alongside resources and images in order to ensure that children develop their conceptual understanding of the written method being taught.
- It should also be made clear that it is not a process that the children use for every type of calculation, but rather chosen when it is not appropriate to complete the calculation mentally or mentally with jotting.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

As such, children should be encouraged to:

- Look at a calculation and decide whether it can be done mentally, mentally with a jotting or whether it needs a written method.
- Estimate, calculate and check to ensure that the answer they generate has some meaning.

The policy also outlines the mental strategies that children should be encouraged to use. These generally fall into one of two categories:

- A mental strategy that they can always rely on (e.g. counting in tens and ones, forwards and backwards, for example 56 – 25 would involve counting back in 10s - 56, 46, 36 - and then back in ones - 36, 35, 34, 33, 32, 31)
- A special strategy they can select if they can see something special about the numbers they are being asked to calculate with (e.g. using near doubles to solve 46 – 24, by working out 46 – 23 – 1)

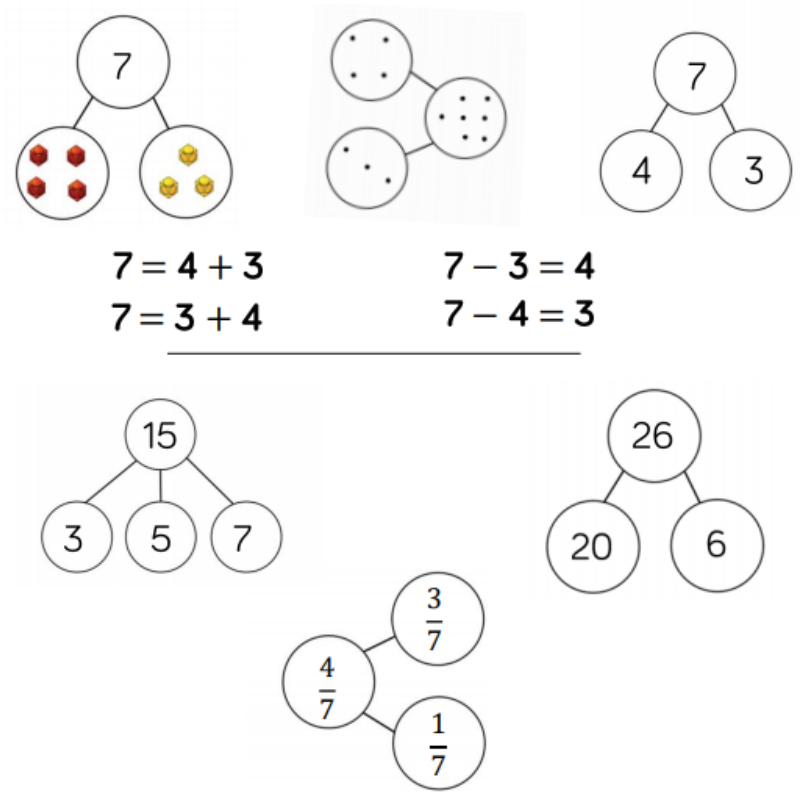
Note: In order to develop understanding of the = sign, its position should be varied in number sentences/equations and empty box/missing number problems utilised when teaching calculations from Year 1 onwards.

11	Bead Strings	50	Base 10/Dienes (multiplication)
12	Number Tracks	51	Base 10/Dienes (division)
13	Number Lines (labelled)	52	Place value counters (multiplication)
14	Number lines (blank)	53	Place value counters (division)
15	Straws	54	**Multiplication (Key Vocabulary & Mental Strategies)**
16	Base 10/Dienes (addition)	55 & 56	Stage 1: Year 1/2 objectives. Skill: Solve 1-step problems using multiplication
17	Base 10/Dienes (subtraction)	57 & 58	Stage 2: Year 3/4 objectives. Skill: Multiply 2-digit numbers by 1-digit numbers
18	Place Value Counters (addition)	59	Stage 3: Year 3/4 objectives. Skill: Multiply 3-digit numbers by 1-digit numbers
19	Place Value Counters (subtraction)	60	Stage 4: Year 5 objective. Skill: Multiply 4-digit numbers by 1-digit numbers
20	Partitioning	61	Stage 5: Year 5 objective. Skill: Multiply 2-digit numbers by 2-digit numbers
21	Same concept, different representations	62	Stage 6: Year 5 objective. Skill: Multiply 3-digit numbers by 2-digit numbers
22	**Addition (Key Vocabulary & Mental Strategies)**	63	Stage 7: Year 5/6 objectives. Skill: Multiply 4-digit numbers by 2-digit numbers
23	Stage 1: Year 1 objective. Skill: Add 1-digit numbers within 10	64	**Division (Key Vocabulary & Mental Strategies)**
24	Stage 2: Year 1/2 objectives. Skill: Add 1 and 2-digit numbers to 20	65	Stage 1: Year 1/2 objectives. Skill: Solve 1-step problems using division (sharing)
25	Stage 3: Year 2 objective. Skill: Add three 1-digit numbers	66	Stage 2: Year 1/2 objectives. Skill: Solve 1-step problems using division (grouping)
26	Stage 4. Year 2/3 objectives. Skill: Add 1-digit and 2-digit numbers to 100	67	Stage 3: Year 1/2 objectives. Skill: Divide 2-digit numbers by 1-digit (sharing no exchange)
27 & 28	Stage 5: Year 2/3 objectives. Skill: Add two 2-digit numbers to 100	68 – 72	Stage 4: Year 3/4 objectives. Skill: Divide 2-digits by 1-digit (sharing with exchange)
29 & 30	Stage 6: Year 3 objective. Skill: Add numbers with up to 3 digits	73	Stage 5: Year 3/4 objectives. Skill: Divide 2-digit numbers by 1-digit (sharing with remainders)
31	Stage 7: Year 4 objective. Skill: Add numbers with up to 4 digits	74	Stage 6: Year 4/5 objectives. Skill: Divide 2-digits by 1-digit (grouping)
32	Stage 8: Year 5/6 objectives. Skill: Add numbers with more than 4 digits	75	Stage 7: Year 4 objective. Skill: Divide 3-digit by 1-digit (sharing)
33	Stage 9: Year 5/6 objectives. Skill: Add decimals with up to 3 decimal places	76	Stage 8: Year 5 objective. Skill: Divide 3-digits by 1-digit (grouping)
34	**Subtraction (Key Vocabulary & Mental Strategies)**	77	Stage 9: Year 5 objective. Skill: Divide 4-digits by 1-digit (grouping)
35	Stage 1: Year 1 objective. Skill: Subtract 1-digit numbers within 10	78	Stage 10: Year 6 objective. Skill: Divide multi-digits by 2-digits (short division)
36	Stage 2: Year 1/2 objectives. Skill: Subtract 1 and 2-digit numbers to 20	79	Stage 11: Year 6 objective. Skill: Divide multi-digits by 2-digits (long division)
37 & 38	Stage 3: Year 2 objective. Skill: Subtract 1 and 2-digit numbers to 100	80	Stage 12: Year 6 objective. Skill: Divide multi-digits by 2-digits (long division)
39	Stage 4: Year 3 objective. Skill: Subtract numbers with up to three digits		
40	Stage 5: Year 4 objective. Skill: Subtract numbers with up to 4 digits		
41	Stage 6: Year 5/6 objectives. Skill: Subtract numbers with more than 4 digits		

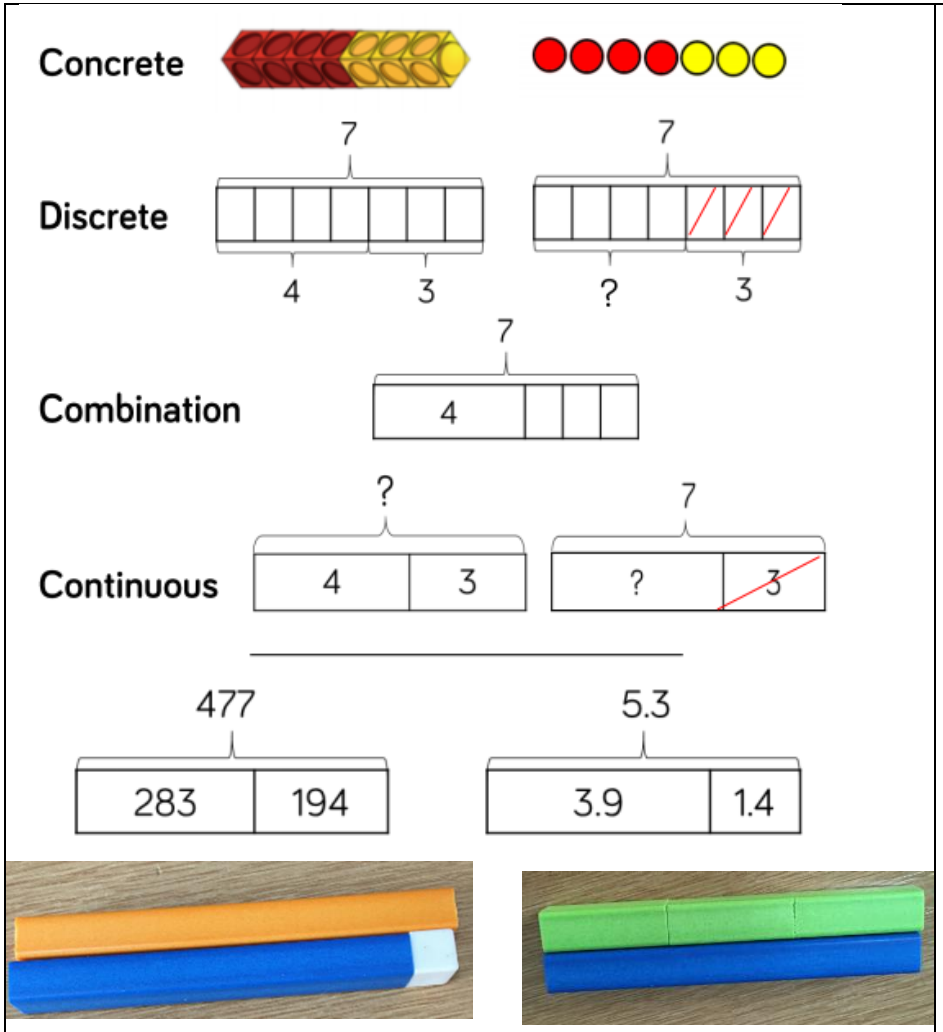
Overview of the different models – addition and subtraction

Part-Whole Model	**Key points** • Supports children in their understanding of partitioning and aggregation.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

	• Children use aggregation to add the parts together to find the total when the parts are complete and the whole is empty. • Children use partitioning to find the missing part when the whole is complete and at least one of the parts is empty. • Part-whole models can be used to partition a number into two or more parts • In KS2, children can use the part-whole model to add and subtract percentages, decimals and fractions.

Bar Model (single)	**Key points** • Cuisenaire rods, cubes and counters and can be used in a line as a concrete representation of the bar model.

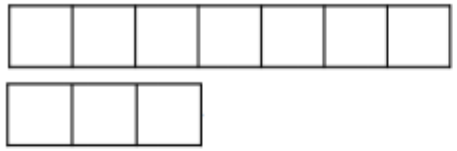


- Can be used as another type of part-whole model to support children in representing calculations.
- Discrete bar models – with each box representing one whole – are a good starting point with smaller numbers
- Combination bar models can support children to count on from the larger number.
- Continuous bar models can be used for a range of values and the question mark indicates the missing value.
- In KS2, bar models can be used to represent larger numbers, decimals and fractions
- Bar models can be used to support children when solving word problems. Children can draw a bar model and use it to represent what is known and unknown in the problem. They then use an appropriate method to solve the problem.

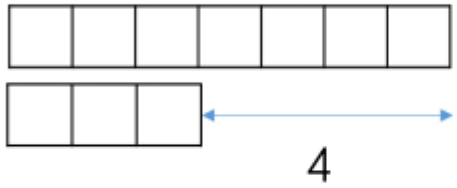
Bar Model (multiple)	**Key points**

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Discrete

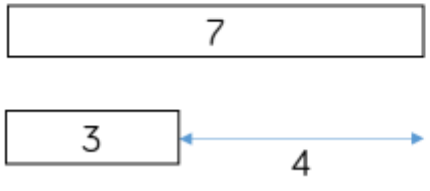


$7 + 3 = 10$

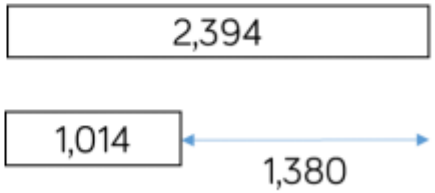


$7 - 3 = 4$

Continuous



$7 - 3 = 4$

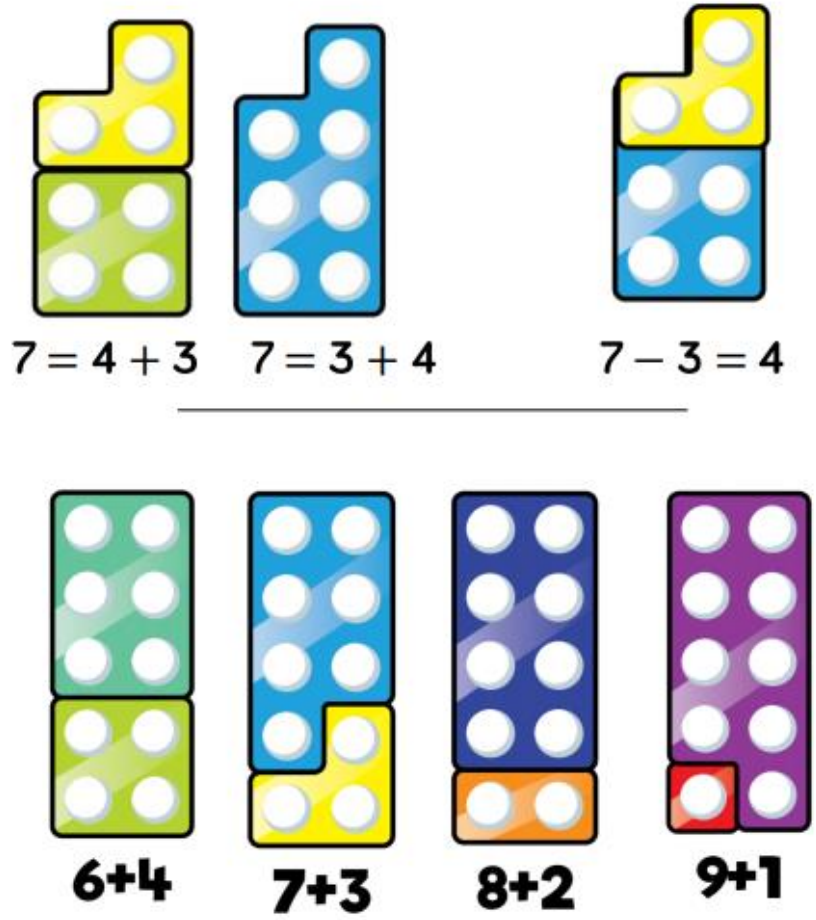


$2{,}394 - 1{,}014 = 1{,}380$

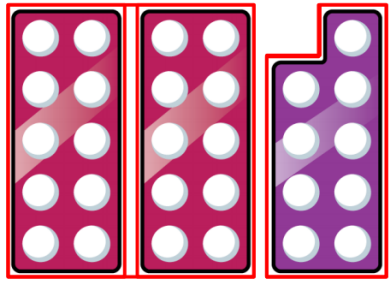
- Children can use cubes and a discrete model to find the difference when working with smaller numbers.
- Smaller numbers can be represented with a discrete bar model.
- Larger numbers can be represented with a continuous bar model
- Multiple bar models can be used to represent the difference in subtraction. An arrow can be used to show the difference.

Numicon	Key points

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



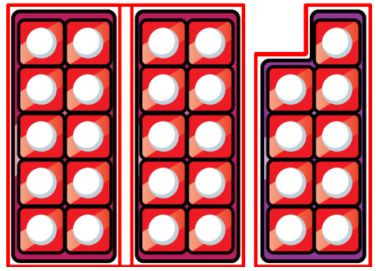
- A great resource to help children subitise numbers as well as explore aggregation, partitioning and number bonds.
- When adding, children can see how the parts come together to make a whole.
- When subtracting, children can start with the whole and then place one of the parts on top of the whole to see what part is missing.
- Children can work systematically to find number bonds.



$29 = 10 + 10 + 9$

$29 = 2$ tens $+ 9$ ones

$29 = 20 + 9$



$29 = 10 + 10 + 9$

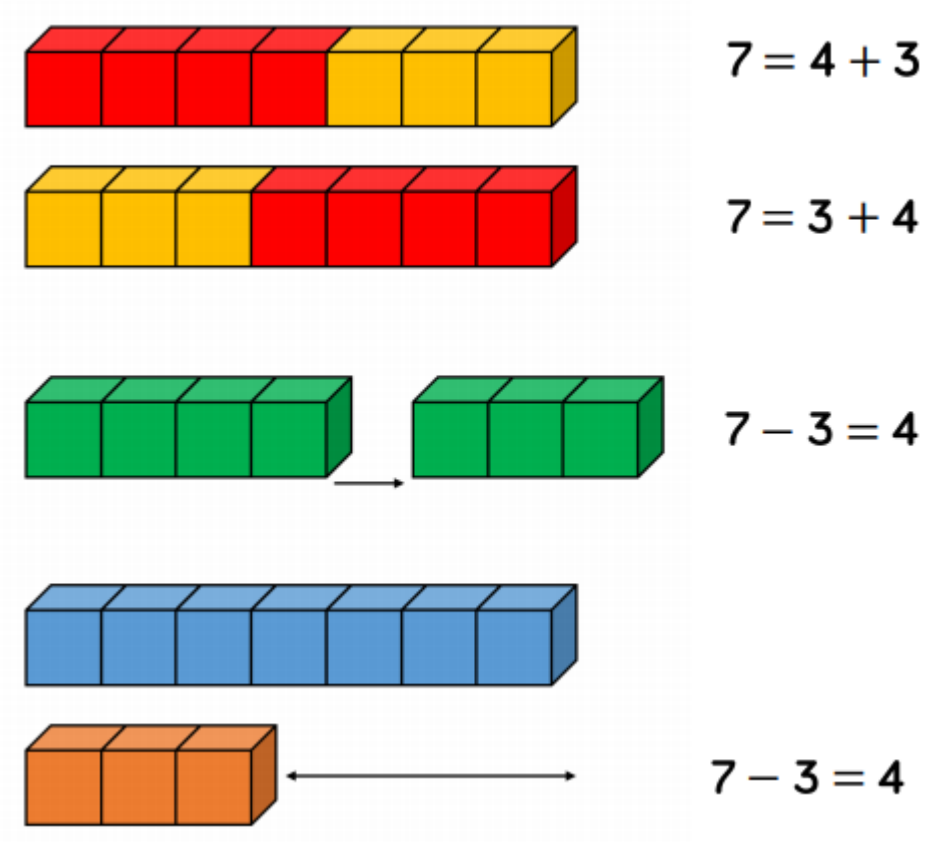
$29 = 2$ tens $+ 9$ ones

$29 = 20 + 9$

$29 = 29$ ones

Cubes	**Key points**

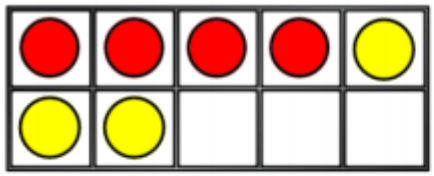
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



- Useful when adding and subtracting one-digit numbers.
- When adding, children can see how the parts come together to make a whole.
- Children can use two different colours to represent the two parts before joining them to create the whole.
- When subtracting, children can start with the whole and then remove the number of cubes that they are subtracting to find the answer. This model of subtraction is reduction or take away.
- To find the difference, both numbers can be made up and then lined next to each other.
- Useful when working with smaller numbers.

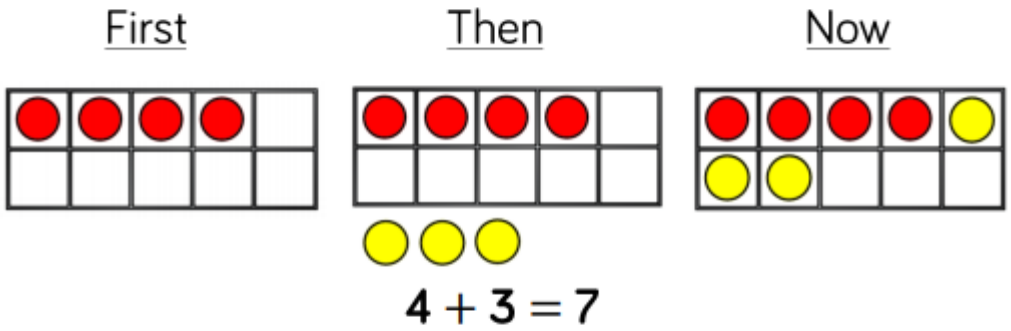
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

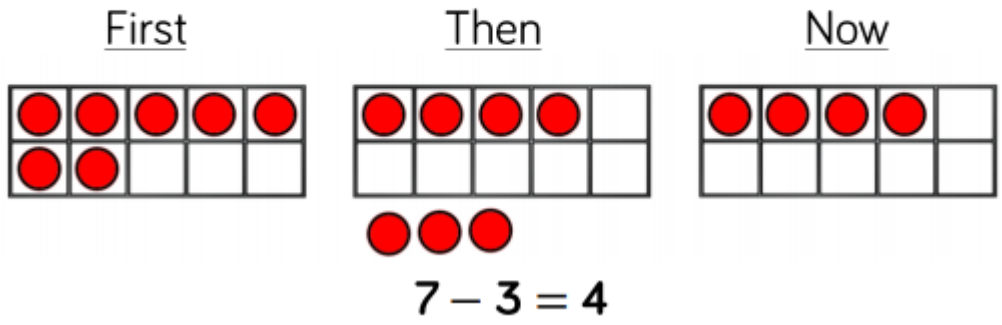
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.



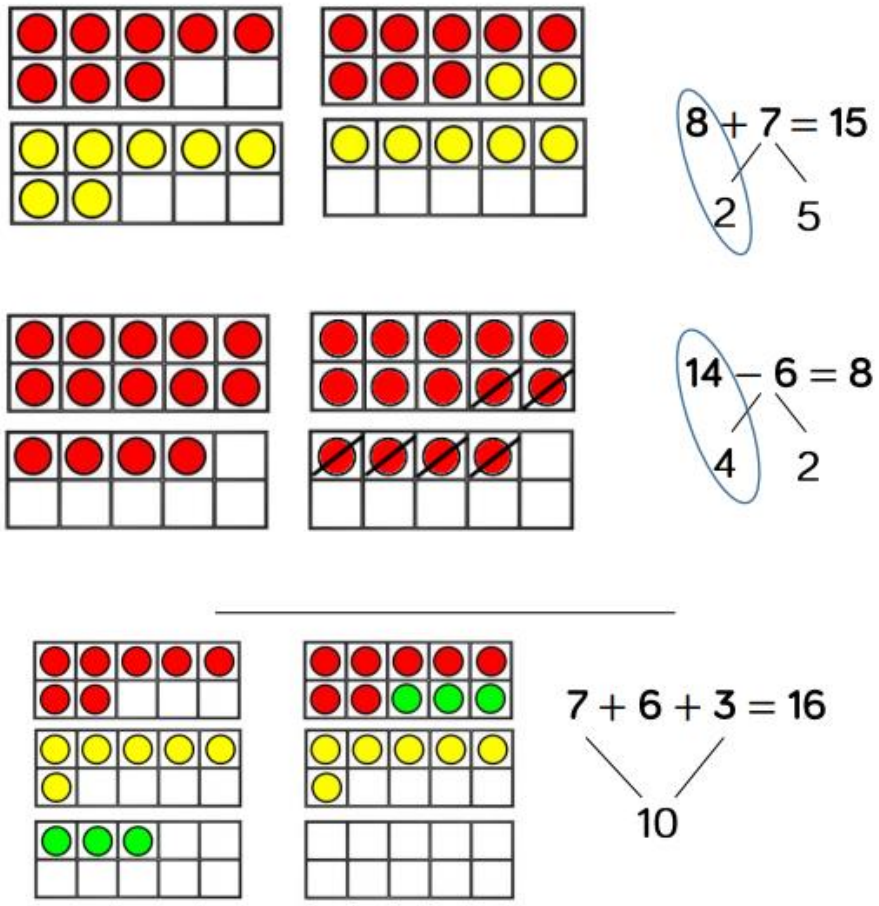


Key points

- When adding and subtracting, the ten frame can support children to understand the different structures of addition and subtraction.
- Linking parts and wholes to the items on the ten frame introduces children to aggregation and partitioning.
- Augmentation and take-away can be represented on a tens frame. As shown by the first, then and now stages.
- Adding a story structure can help children understand the change.
- First there were 7 apples. Then, 3 apples were eaten. Now, there are 4 apples left.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Ten Frames (within 20)

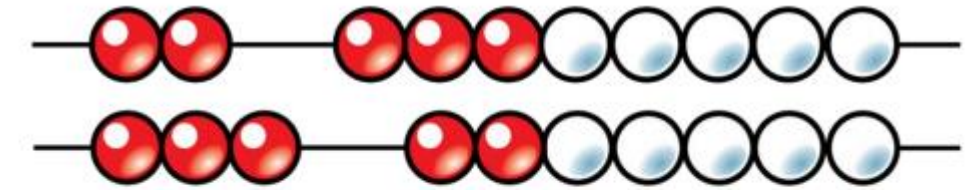


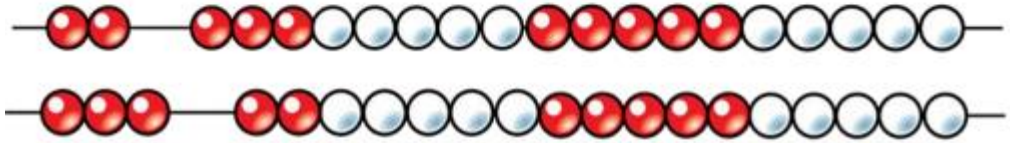
Key points

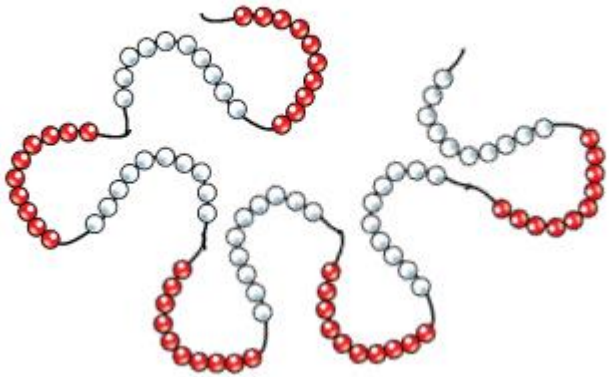
- When adding two single digits, children can make each number on a separate tens frame before moving part of one number to make 10.
- This shows children how they have partitioned one of the numbers to make 10, which makes links to effective mental methods of addition.
- When subtracting, firstly make the larger number on 2 ten frames. Then, remove the smaller number and think how you have partitioned the number to make 10. This supports mental methods of subtraction.
- When adding three single-digit numbers, children can make each number on a separate ten frame.
- Then, they can look to see if they can make a number bond to 10, which would make the calculation easier.
- Here, the ten frames support mental methods of addition and commutativity.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Bead Strings







Key points

10 bead strings

- Effective at helping children investigate number bonds to 10.
- Moving one bead at a time allows children to systematically find all the number bonds to 10, whilst also linking to partitioning. 2 + 8 = 10. 3 + 7 = 10.

20 bead strings

- Group beads into fives.
- Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20

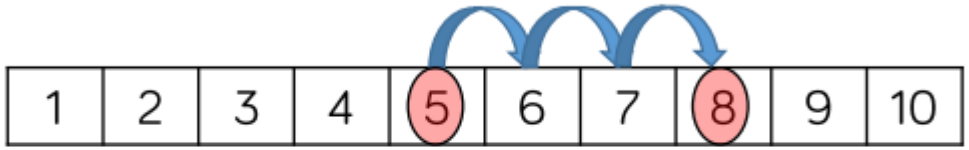
100 bead strings

- Grouped in tens.
- Support number bonds to 100.
- Offer support when adding by making 10.
- Provide a link to adding to the next ten on number lines, which supports a mental method of addition.

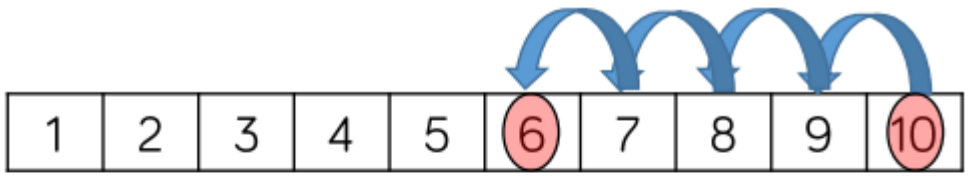
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number Tracks

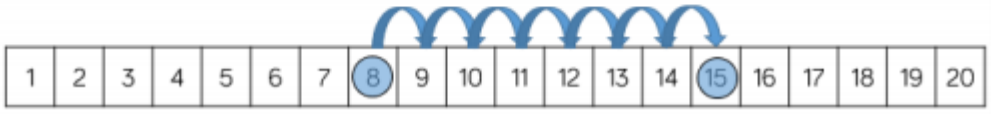
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$

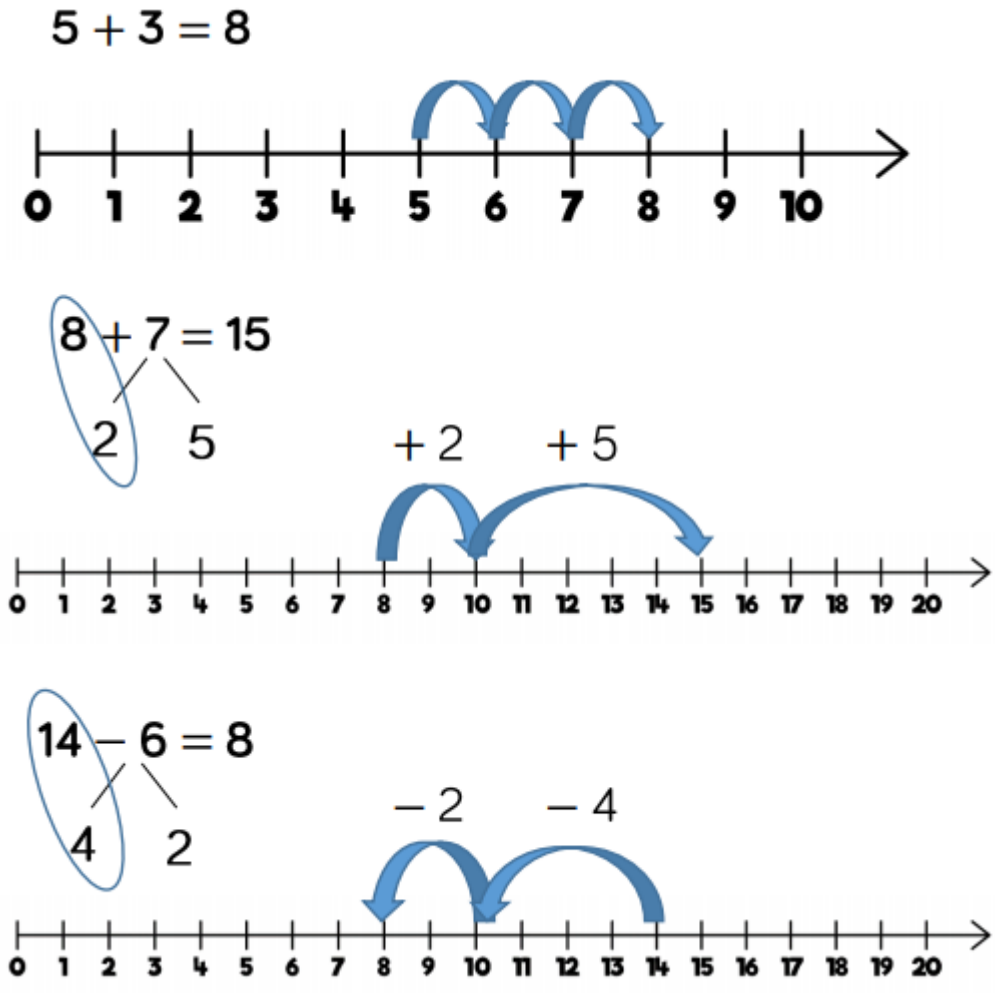


Key points

- Supports children's understanding of augmentation and reduction.
- When adding, children count on to find the total.
- Children can place a counter on the starting number and then count on to find the total.
- When subtracting, children count back to find their answer.
- They start at the minuend and then take away the subtrahend to find the difference.
- Work well alongside ten frames and bead strings as they all model counting on or counting back.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number Lines (labelled)



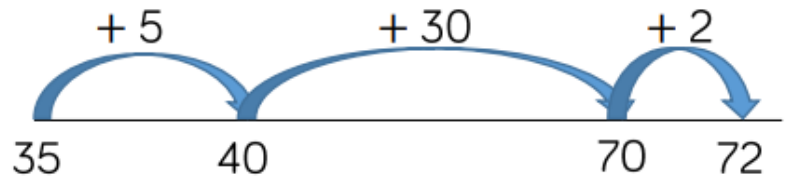
Key points

- Support children in their understanding of addition and subtraction as augmentation and reduction.
- Start by counting on or back in ones (up or down the number line).
- Links directly to the use of a number track.
- Children can add numbers by jumping to the nearest 10 and then jumping to the total.
- This links to the making 10 method (also supported by ten frames).
- The smaller number is partitioned to support children when making a number bond to 10.
- Then, the remaining part is added.
- Children can subtract numbers by jumping to the nearest 10 first (supported by a ten frame).
- This shows children how they partition the smaller number into separate jumps.

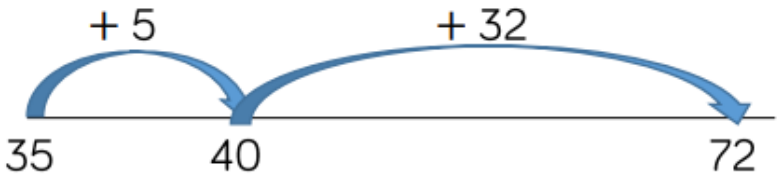
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number Lines (blank)

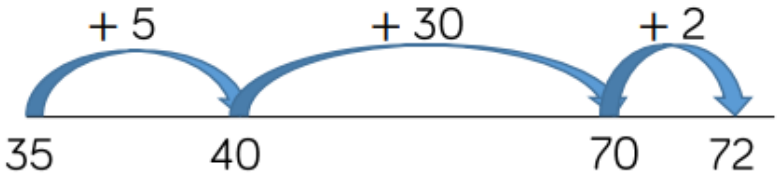
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



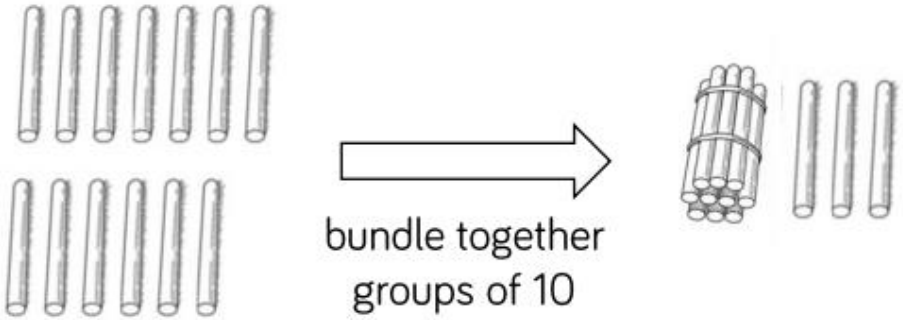
Key points

- Provide a structure for children to add and subtract numbers in smaller parts.
- Children can add by jumping to the nearest 10 and then adding the rest of the number as a whole or by adding tens and ones separately.
- Children can also count back on a number line to subtract. Firstly, they jump backwards to the nearest 10 and then subtracting the rest of the number.
- Blank number lines provide an effective way of finding the difference between numbers by counting on.
- Children start at the smallest number and count on to the largest number. They then add the parts they have counted on to find the difference.

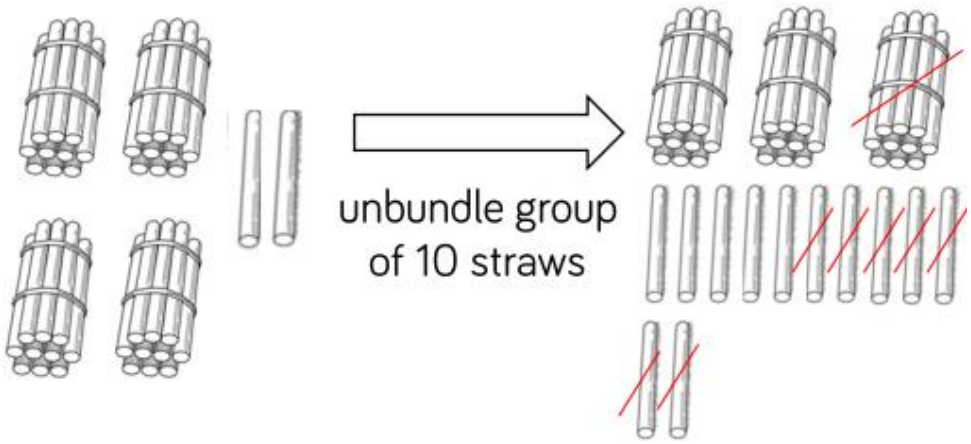
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Straws

$7 + 6 = 13$

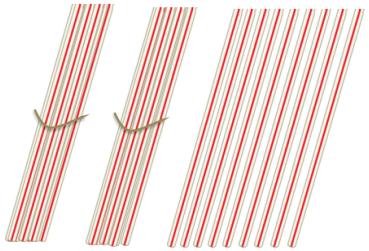


$42 - 17 = 25$



Key points

- Effective way of supporting children to understand exchanging when adding or subtracting 2-digit numbers.
- Introduces the idea of bundling groups of ten.
- When adding, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten.
- They then find the total by adding the bundles (tens) and individual straws (ones).
- When subtracting, children can unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.
- Straws provide a good step towards adding and subtracting with Base 10/Dienes.

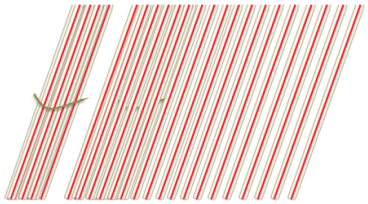


$29 = 10 + 10 + 9$

$29 = 2$ tens $+ 9$ ones

$29 = 20 + 9$

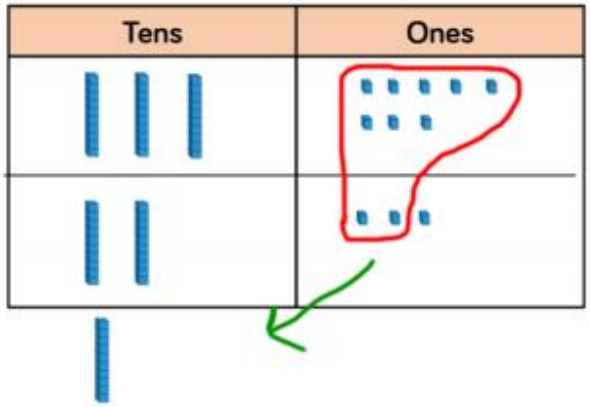
$29 = 29$ ones



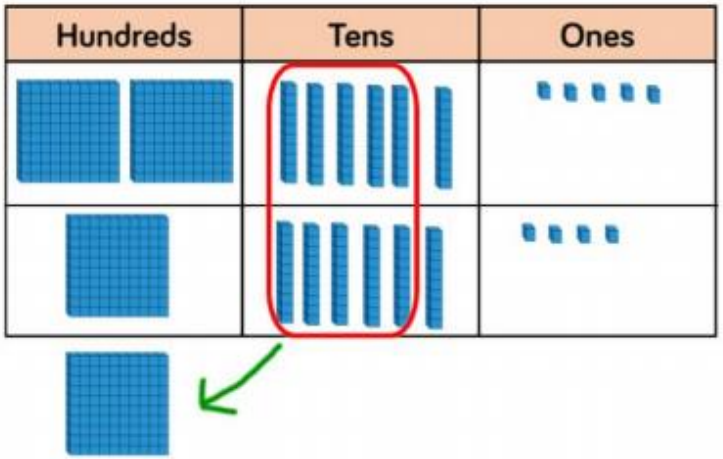
$29 = 2$ tens $+ 9$ ones
$29 = 20 + 9$
$29 = 10 + 10 + 9$
$29 = 29$ ones
$29 = 1$ ten $+19$ ones
$29 = 10 + 19$

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\,23 \\ \hline 61 \\ \hline 1 \end{array}$$



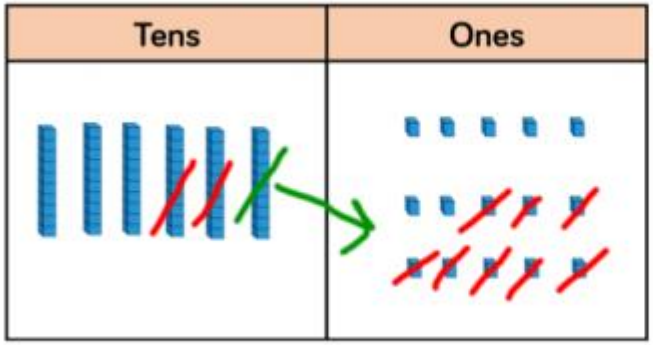
$$\begin{array}{r} 265 \\ +\,164 \\ \hline 429 \\ \hline 1 \end{array}$$

Key points

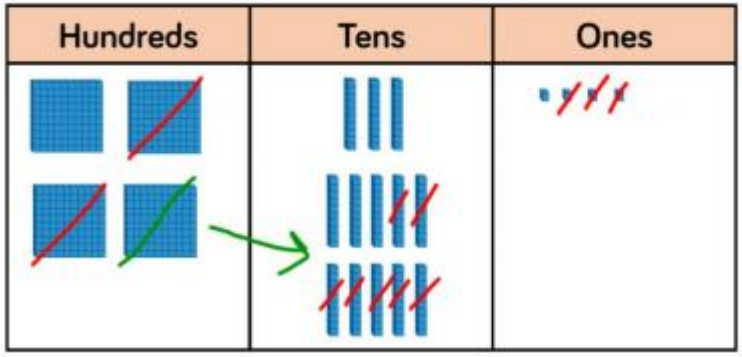
- Provides an effective way to support children's understanding of column addition.
- It is imperative that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the model and the written method.
- Children should first add without exchange.
- This representation becomes less efficient with larger numbers.
- The next step is to use place value counters.
- When adding, always add from the smallest place value column.
- Key questions to ask the children:
- How many ones are there altogether?
- Can we make an exchange? (Yes or No).
- How many do we exchange? (10 ones for 1 ten)
- Show the exchanged 10 in the tens column by drawing and writing 1 below the column.
- How many ones do we have left? (Write the digit in the ones column).
- Repeat for each column.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 28 \\ \hline 37 \end{array}$$



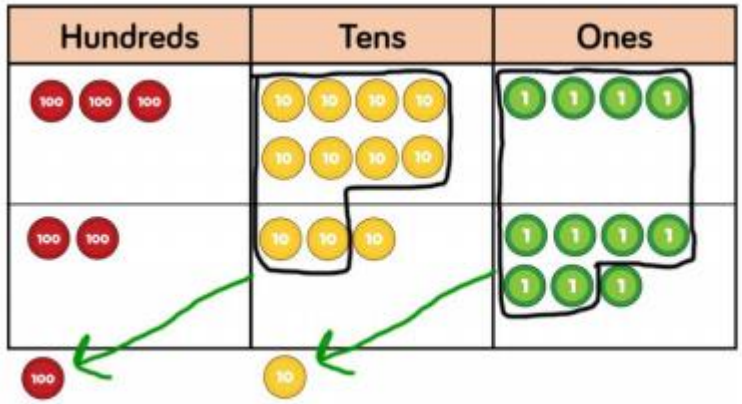
$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}\not{5} \\ -\ 273 \\ \hline 262 \end{array}$$

Key points

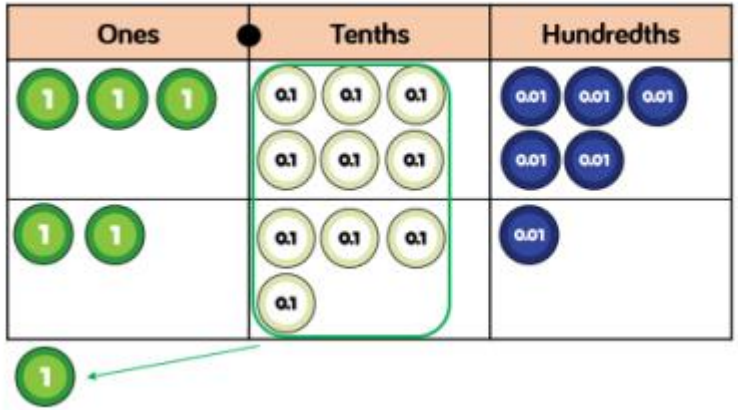
- Provides an effective way to support children's understanding of column subtraction.
- It is imperative that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the model and the written method.
- Children should first subtract without exchange.
- When building the model, children should just make the minuend (a number or quantity from which another number is subtracted) using Base 10.
- Then, they subtract the subtrahend (a number to be subtracted from another).
- Highlight this difference to addition to avoid errors by making both numbers.
- Children start by subtracting from the smallest place value column.
- When there are not enough ones/tens/hundreds to subtract in a column, children need to exchange from the column to the left (1 ten for 10 ones).
- They can then subtract efficiently.
- This model is efficient with up to 4-digit numbers.
- Place value counters are more efficient with larger numbers.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ 1 \end{array}$$



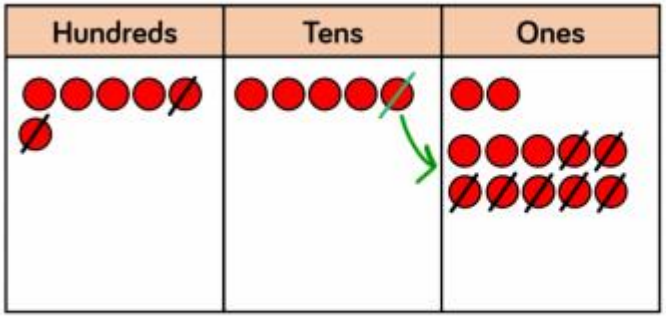
$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

Key points

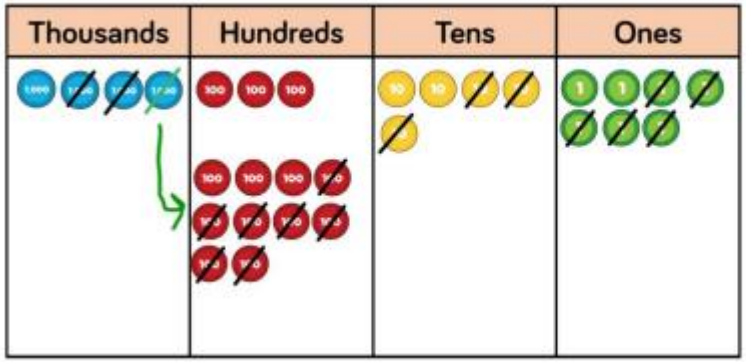
- Provides an effective way to support children's understanding of column addition.
- It is imperative that children write out their calculations alongside using or drawing counters to develop their conceptual understanding.
- First add without exchange.
- Different place value counters can be used to represent larger numbers or decimals.
- Normal counters on a place value grid can also be used to enable children to experience the exchange between columns.
- When adding money, children can also use coins to support their understanding.
- It is important to show children the links between the coins when using the written method to support the addition of decimal amounts.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Place Value Counters (subtraction)



$$\begin{array}{r} {}^{4}\not{6}\,{}^{1}\not{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



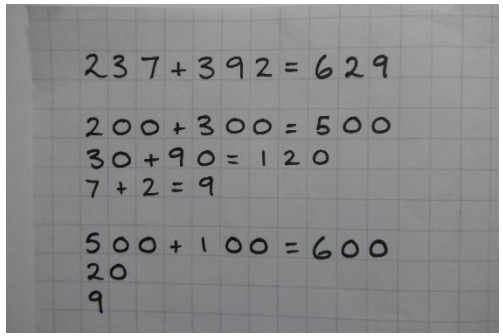
$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

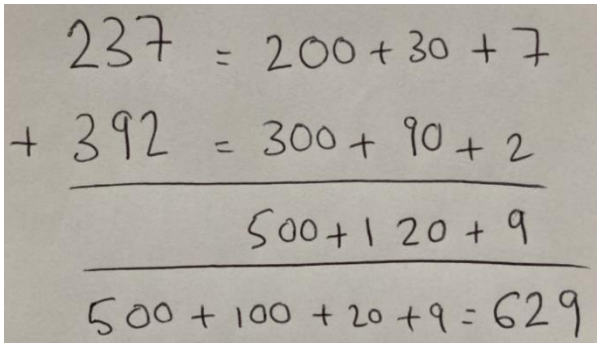
Key points

- Provides an effective way to support children's understanding of column subtraction.
- It is imperative that children write out their calculations alongside using or drawing counters to develop their conceptual understanding.
- First subtract without exchange.
- Normal counters on a place value grid can also be used to enable children to experience the exchange between columns.
- When building the model, children should just make the minuend (a number or quantity from which another number is subtracted) using counters.
- Then, they subtract the subtrahend (a number to be subtracted from another).
- Highlight this difference to addition to avoid errors by making both numbers.
- Children start by subtracting from the smallest place value column.
- When there are not enough ones/tens/hundreds to subtract in a column, children need to exchange from the column to the left (1 ten for 10 ones).
- They can then subtract efficiently.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Partitioning





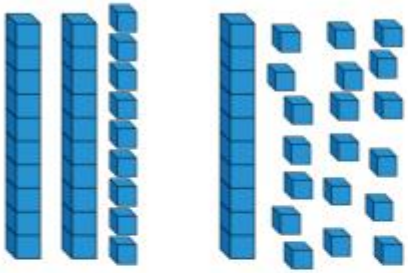
Key points

- Partitioning is a way of splitting numbers into smaller parts to make them easier to work with.
- Partitioning links closely to place value.
- For example, the number 54 represents 5 tens and 4 ones.
- This shows how the number can be partitioned into 50 and 4.
- Partitioning can be used horizontally or vertically as show by the pictures.

Same concept, different representations

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Base 10

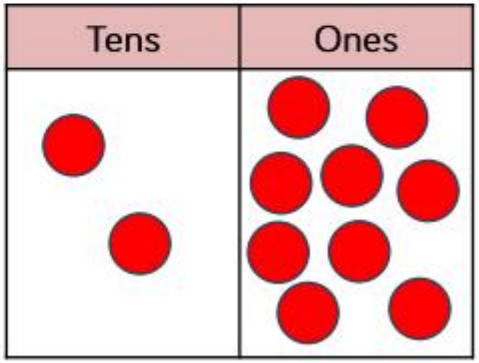


29

Place value counters

Cuisenaire rods

Place value grid



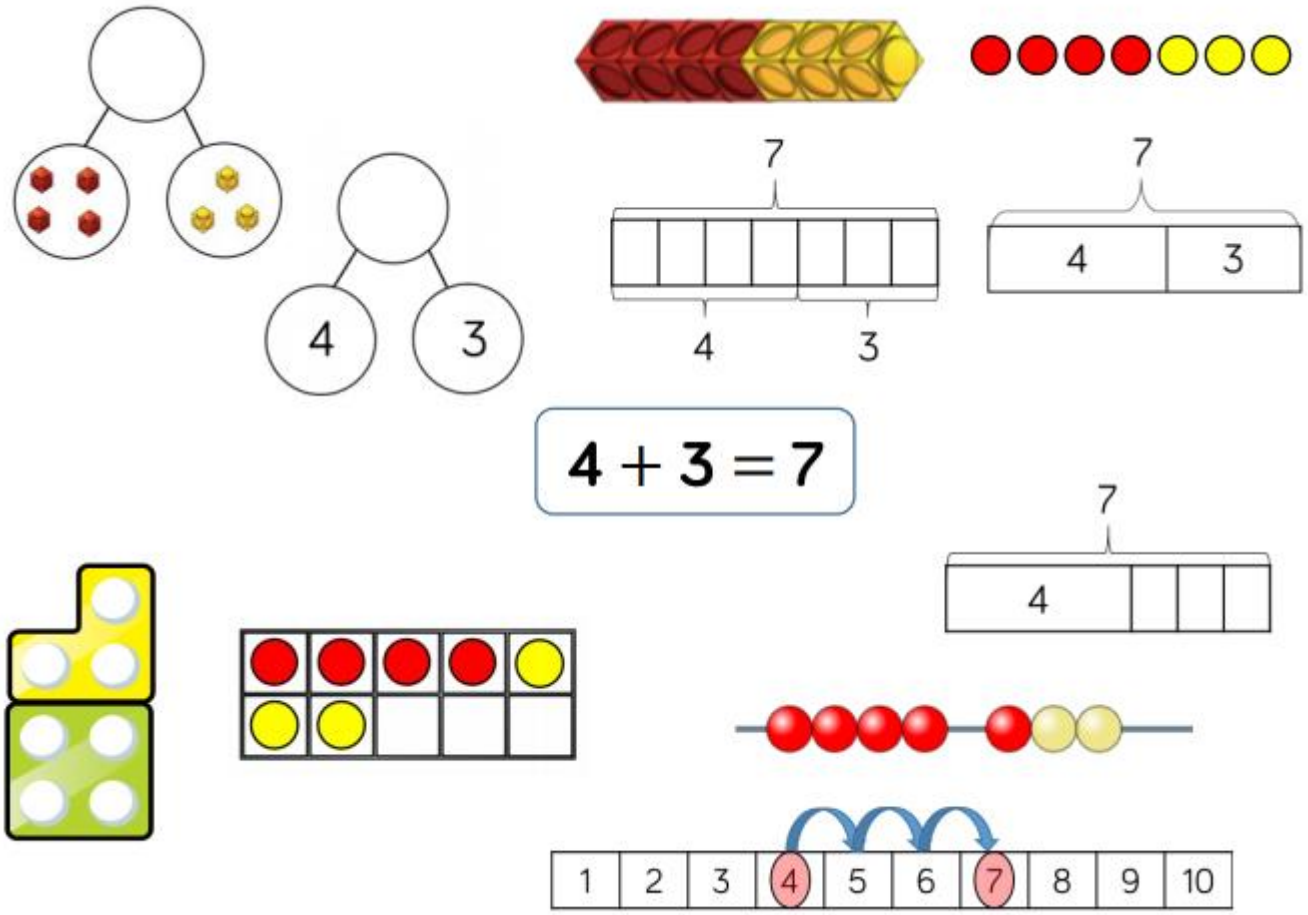
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Addition

Key Vocabulary	
Addend – A number to be added to another Aggregation – combining two or more quantities or measures to find a total Augmentation – increasing a quantity or measure by another quantity Commutative – numbers can be added in any order Exchange – change a number or expression or another of an equal value	Partitioning – splitting a number into its component parts Subitise – instantly recognize the number of objects in a small group without needing to count Sum – the result of addition Total – the aggregate or the sum found by addition
Mental Strategies (see *Teaching Children to Calculate Mentally* for more details and activities to develop these) Counting forwards (e.g. in 1s, 10s, 100s, etc.) Reordering (e.g. adding linked numbers first, such as pairs that make 10, or starting with the larger number, etc.) Partitioning (e.g. adding tens then units, etc.) Bridging through multiples of 10 (e.g. 6 + 7 calculated as 6 + 4 + 3, etc.) Compensating (e.g. 34 + 9 calculated as 34 + 10 - 1, etc.) Near doubles (e.g. 13 + 14 calculated as double 13 + 1, etc.)	

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 1: Year 1 objective



Skill: Add 1-digit numbers within 10

The following models support aggregation:

- Part-whole model
- Discrete bar model
- Continuous bar model
- Numicon
- Ten frame

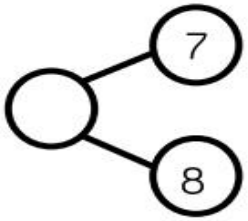
The following models support augmentation:

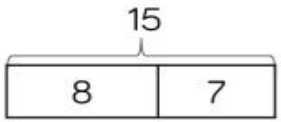
- Combination bar model
- Ten frame
- Bead string
- Number track

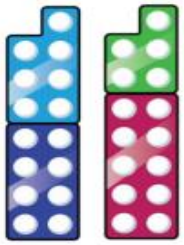
Always write the abstract calculation next to any resources or pictorial representations to support conceptual understanding.

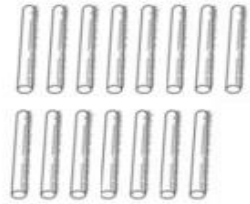
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

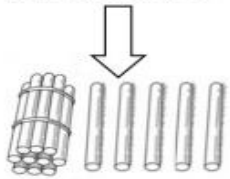
Stage 2: Year 1/2 objectives



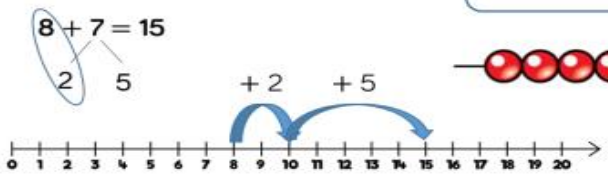


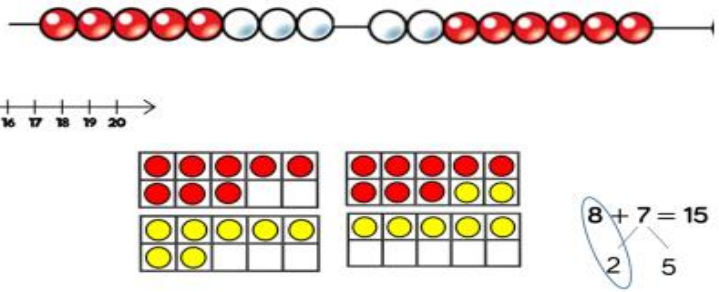


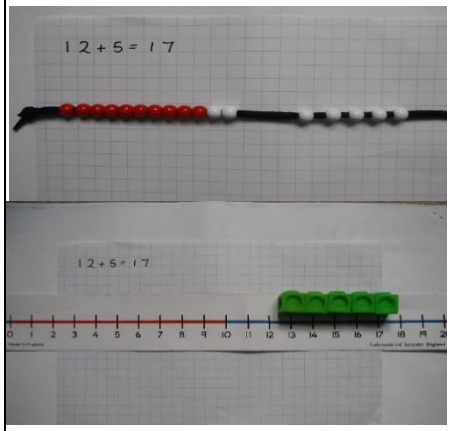


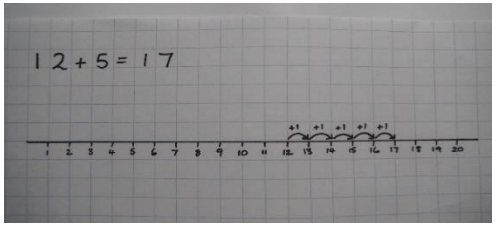


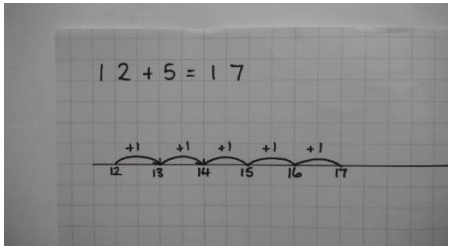
$8 + 7 = 15$











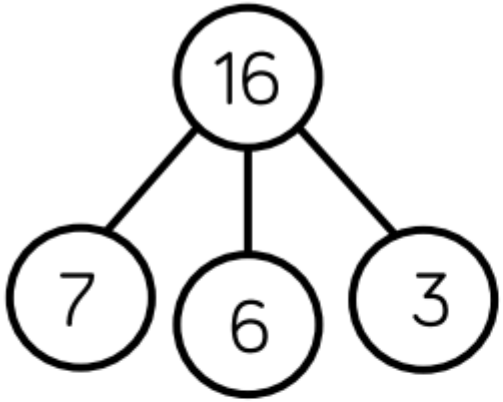
Skill: Add 1 and 2-digit numbers to 20

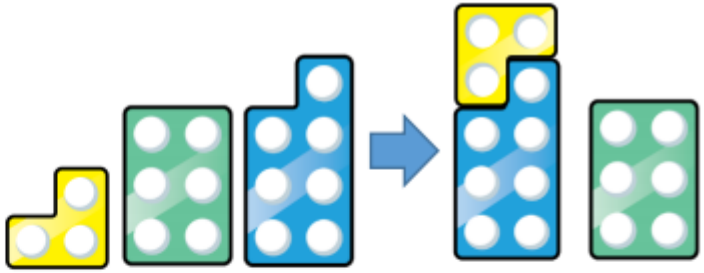
Crossing the tens boundary:

- Use a range of manipulatives to represent this exchange
- Use concrete resources alongside number lines to show children how to partition their jumps.
- Highlight the importance of ten ones equaling one ten

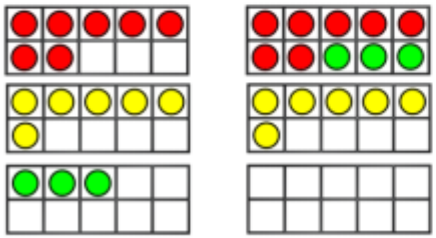
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 3: Year 2 objective



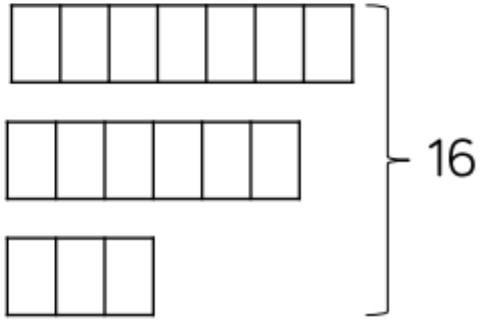


$7 + 6 + 3 = 16$



$7 + 6 + 3 = 16$

10



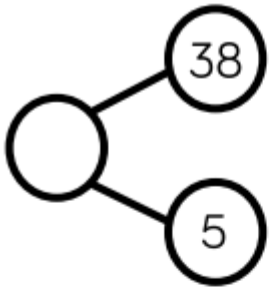
Skill: Add three 1-digit numbers

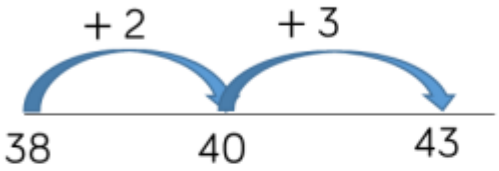
Number bonds to 10

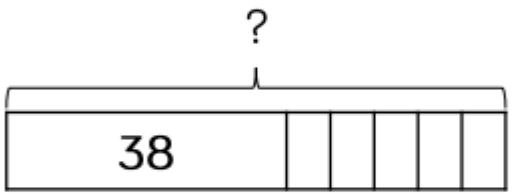
- In order to add the number more efficiently, encourage children to look for number bonds to 10.
- This will also develop their understanding of the commutative law.
- Using concrete resources that highlight number bonds to 10 will support children to add three 1-digit numbers.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

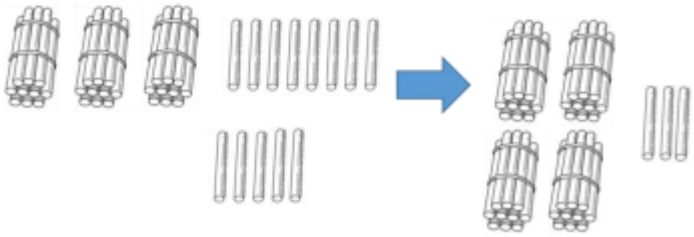
Stage 4: Year 2/3 objectives







$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Skill: Add 1-digit and 2-digit numbers to 100

Encourage children to count on from the largest number.

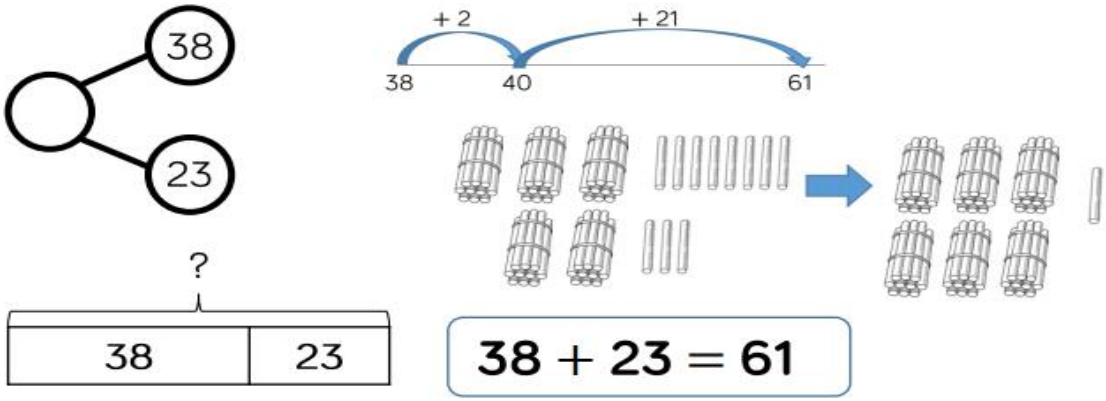
Highlight links to number bonds:

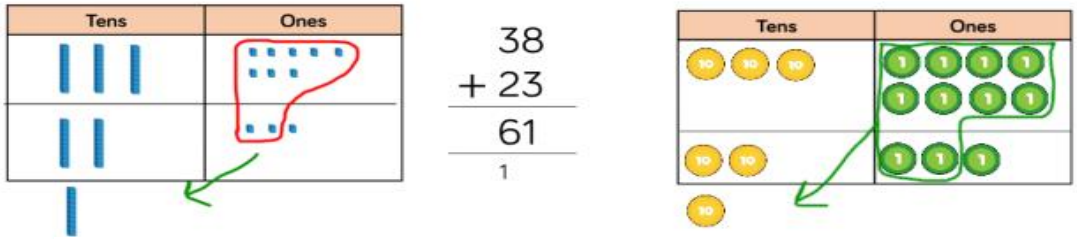
- 8 + 5 = 13
- 38 + 5 = 43

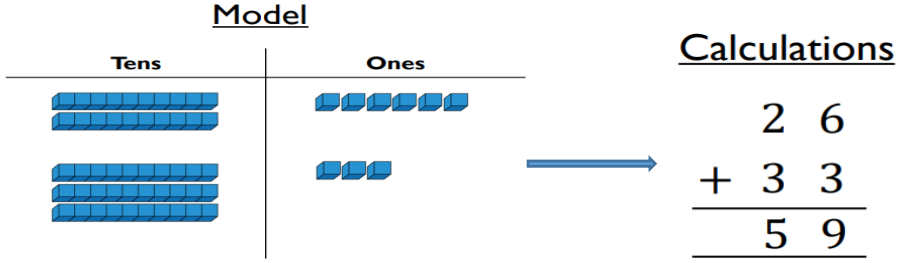
Use straws and hundred squares to help children find number bonds to 10.

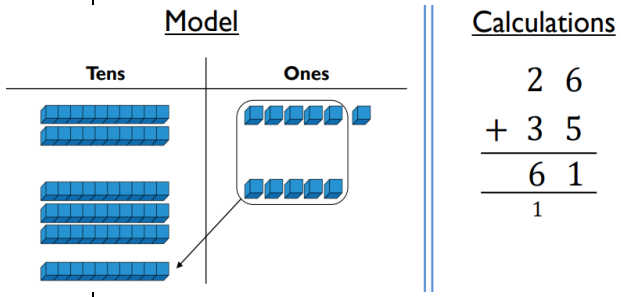
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 5: Year 2/3 objectives









Skill: Add two 2-digit numbers to 100

Move away from straws as the numbers increase.

When using a blank number line, encourage children to jump to multiples of 10.

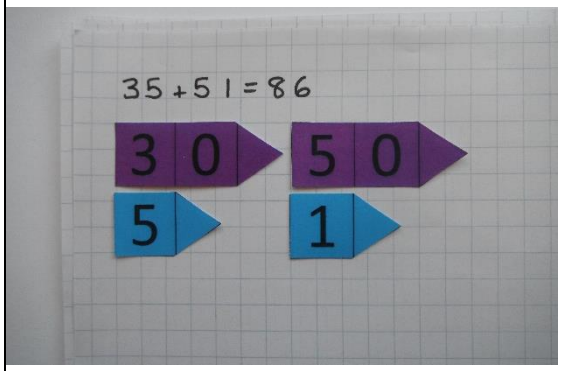
Model the formal column method alongside concrete resources and pictorial representations.

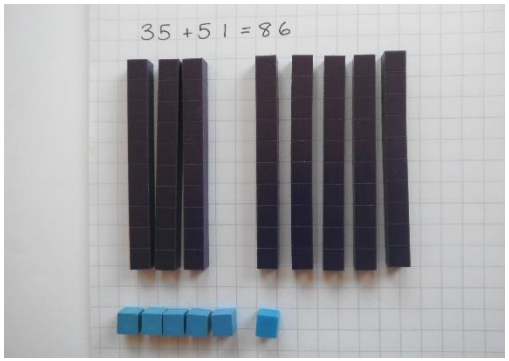
If there is nothing in a place value column, we use 0 as a place holder.

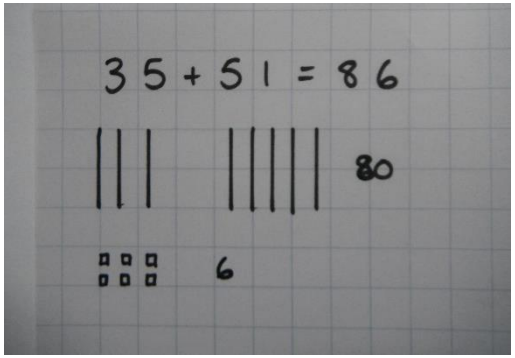
Start from the right hand column and work to the left.

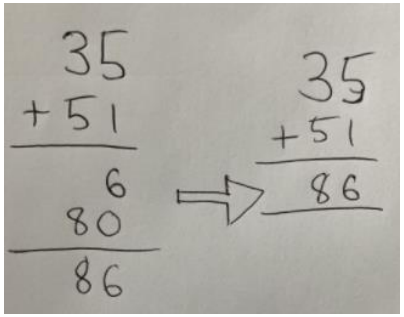
Shows exchange when there are 10 in a column.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



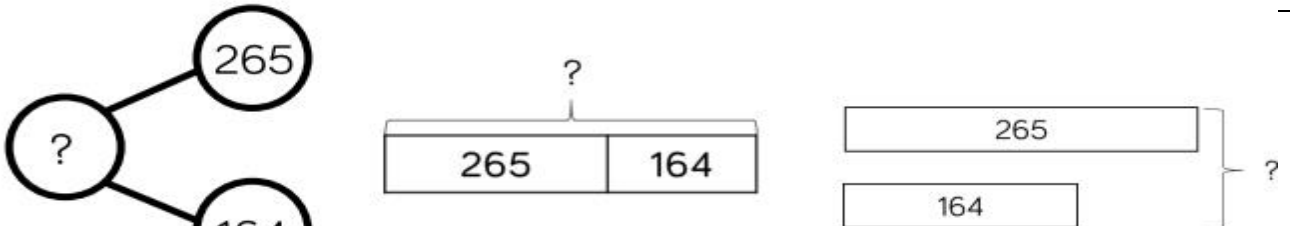




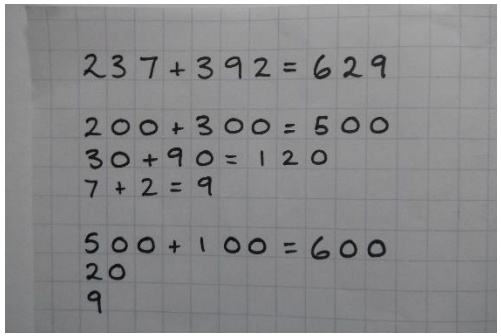


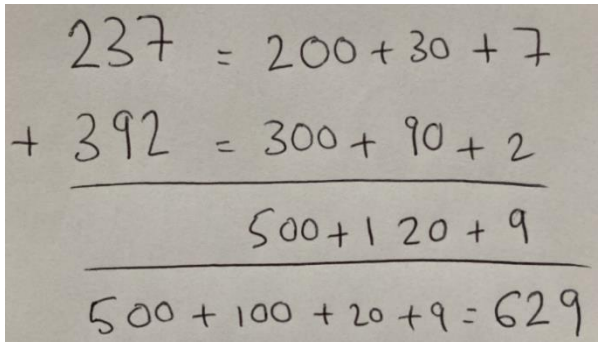
Stage 6: Year 3 objective

Skill: Add numbers with up to 3 digits

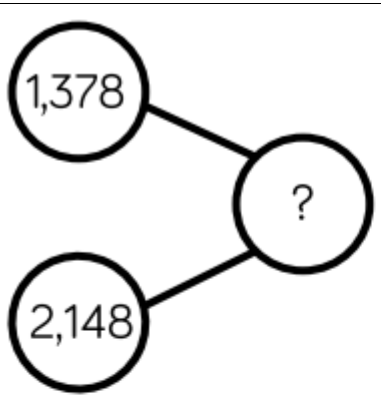


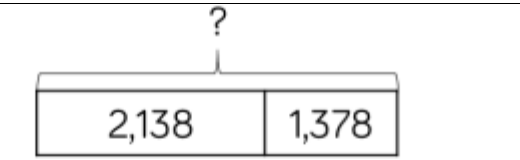
	When adding numbers with up to 3 digits, Base 10 and place value counters are the most effective concrete resources. Always model the concrete resources and/or pictorial representations alongside the column method. Ensure children do the same in order for them to see how the manipulatives link to the formal method. If there is nothing in a place value column, we use 0 as a place holder. Start from the right hand column and work to the left. Shows exchange when there are 10 in a column.

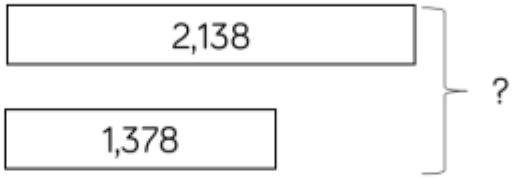




Stage 7: Year 4 objective	**Skill: Add numbers with up to 4 digits**

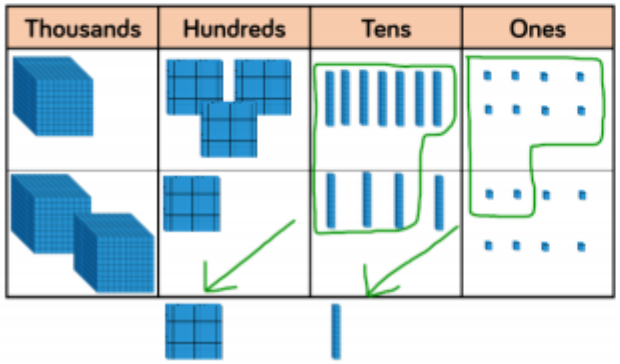






	1	3	7	8
+	2	1	4	8
	3	5	2	6
	1	1		

1,378 + 2,148 = 3,526



When adding numbers with up to 4 digits, Base 10 and place value counters are the most effective concrete resources.

Always model the concrete resources and/or pictorial representations alongside the column method.

Ensure children do the same in order for them to see how the manipulatives link to the formal method.

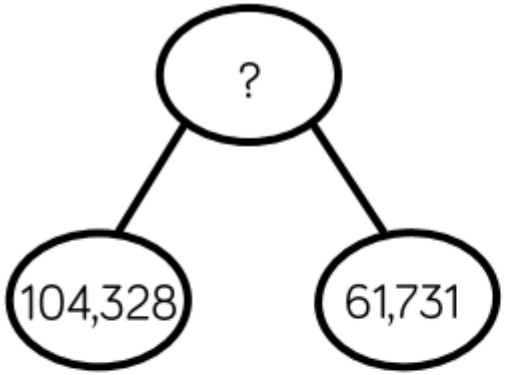
If there is nothing in a place value column, we use 0 as a place holder.

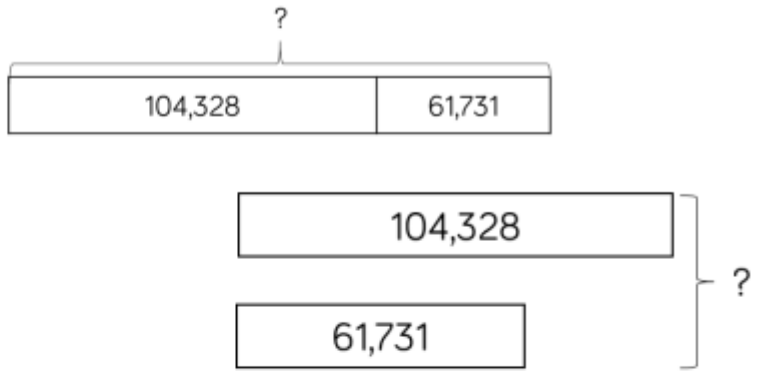
Start from the right hand column and work to the left.

Shows exchange when there are 10 in a column.

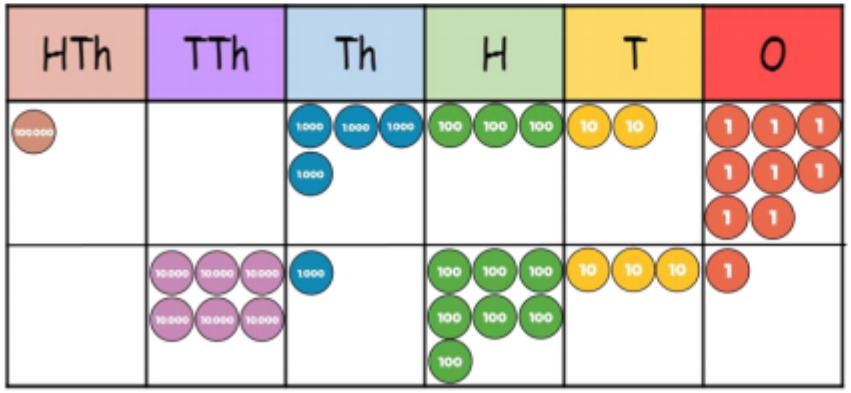
Stage 8: Year 5/6 objectives	**Skill: Add numbers with more than 4 digits**

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy





$$104{,}328 + 61{,}731 = 166{,}059$$



1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

1

When adding numbers with more than 4 digits, place value counters or plain counters are the most effective concrete resources.

Always model the concrete resources and/or pictorial representations alongside the column method. Ensure children do the same in order for them to see how the manipulatives link to the formal method.

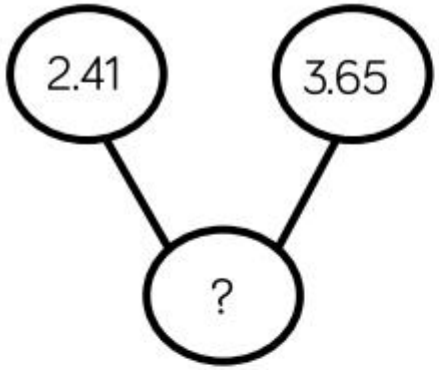
Encourage children to work in the abstract form by using the column method to efficiently add large numbers.

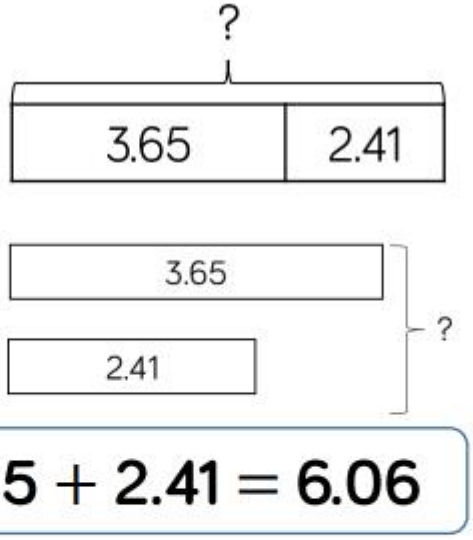
If there is nothing in a place value column, we use 0 as a place holder.

Start from the right hand column and work to the left.
Shows exchange when there are 10 in a column.

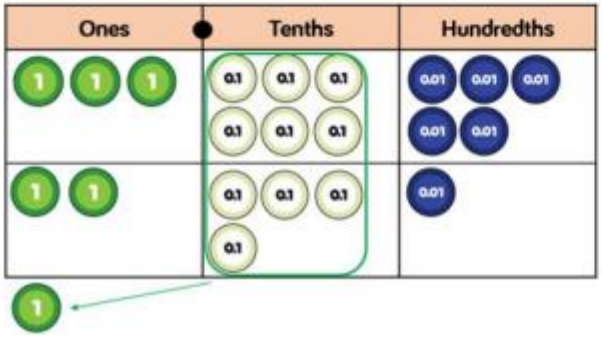
Stage 9: Year 5/6 objectives	**Skill: Add decimals with up to 3 decimal places**

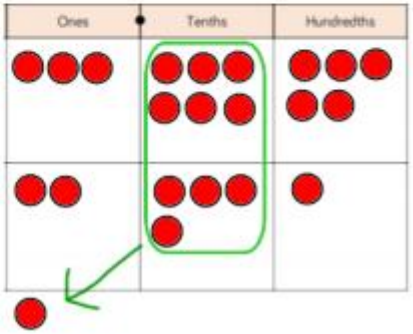
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy





$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$





When adding decimals with up to 3 decimal places, place value counters or plain counters on a place value grid are the most effective concrete resources.

Always model the concrete resources and/or pictorial representations alongside the column method. Ensure children do the same in order for them to see how the manipulatives link to the formal method.

Encourage children to work in the abstract form by using the column method to efficiently add large numbers.

Adding money should be used at this stage to add context.

If there is nothing in a place value column, we use 0 as a place holder.

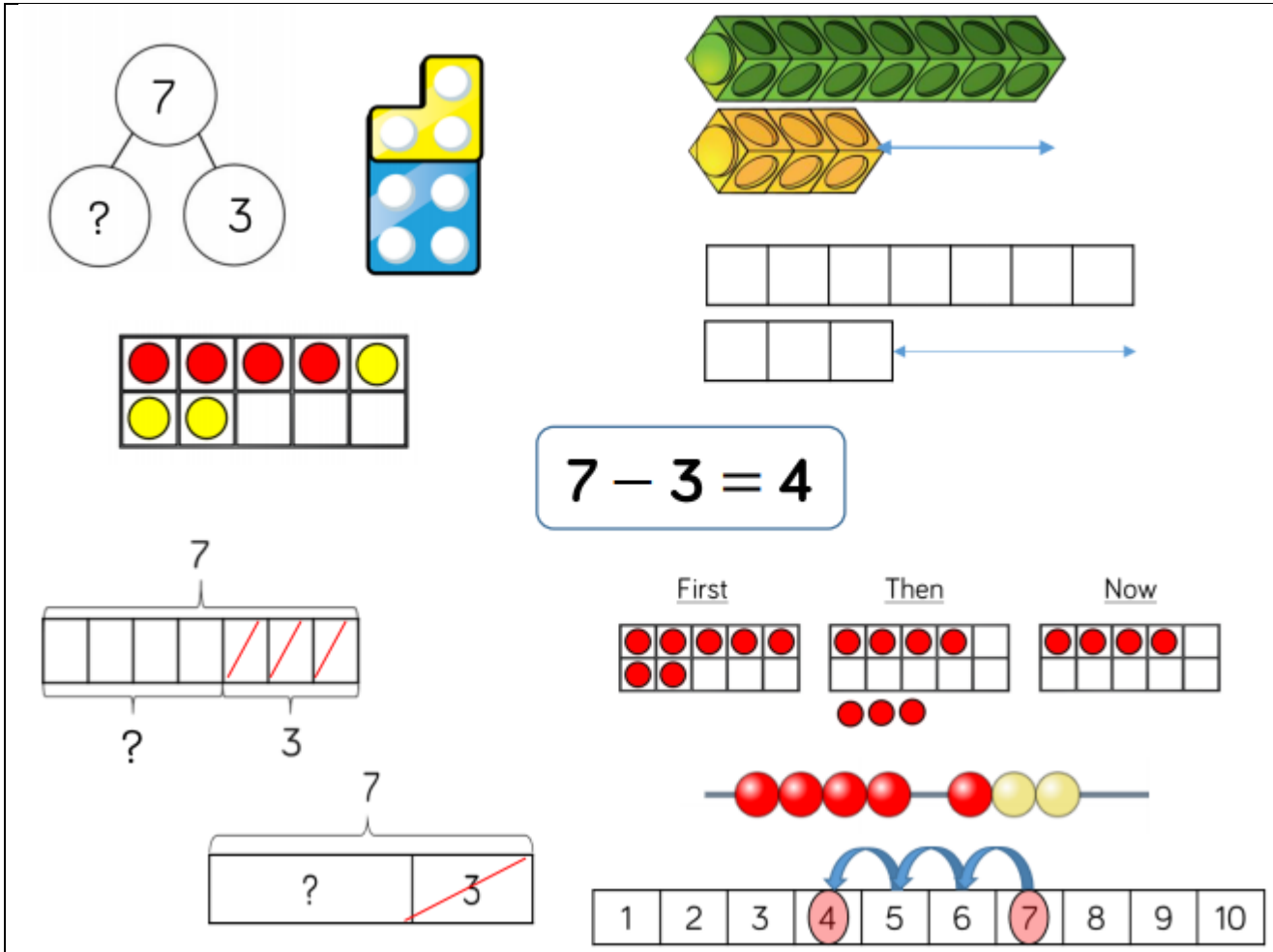
Start from the right hand column and work to the left. Shows exchange when there are 10 in a column.

Subtraction

Key Vocabulary	

Difference – the numerical difference between two numbers is found by comparing the quantity in each group Exchange – change a number or expression or another of an equal value Minuend – a quantity or number from which another is subtracted	Partitioning – splitting a number into its component parts Subitise – instantly recognize the number of objects in a small group without needing to count Reduction – subtraction as take away Subtrahend – a number to be subtracted from another
Mental Strategies (see *Teaching Children to Calculate Mentally* for more details and activities to develop these) Counting backwards (e.g. in 1s, 10s, 100s, etc.) Counting forwards to find the difference (e.g. in 1s, 10s, 100s, etc.) Reordering (e.g. 12 - 7- 2 can be reordered into 12 - 2 - 7, etc.) Partitioning (e.g. subtracting tens then units, etc.) Bridging through multiples of 10 (e.g. 12 - 7 calculated as 12 - 2 - 5, or 607 - 288 calculated as 288 + 12 + 300 + 7) Compensating (e.g. 70 - 9 calculated as 70 - 10 + 1, or 405 - 399 calculated as 405 - 400 + 5, etc.)	

Stage 1: Year 1 objective	**Skill: Subtract 1-digit numbers within 10** The following models support partitioning:



- Part-whole model
- Bar model
- Ten frame
- Numicon

The following models support reduction:

- Ten frame
- Number track
- Singe bar model
- Bead string

The following models support finding the difference:

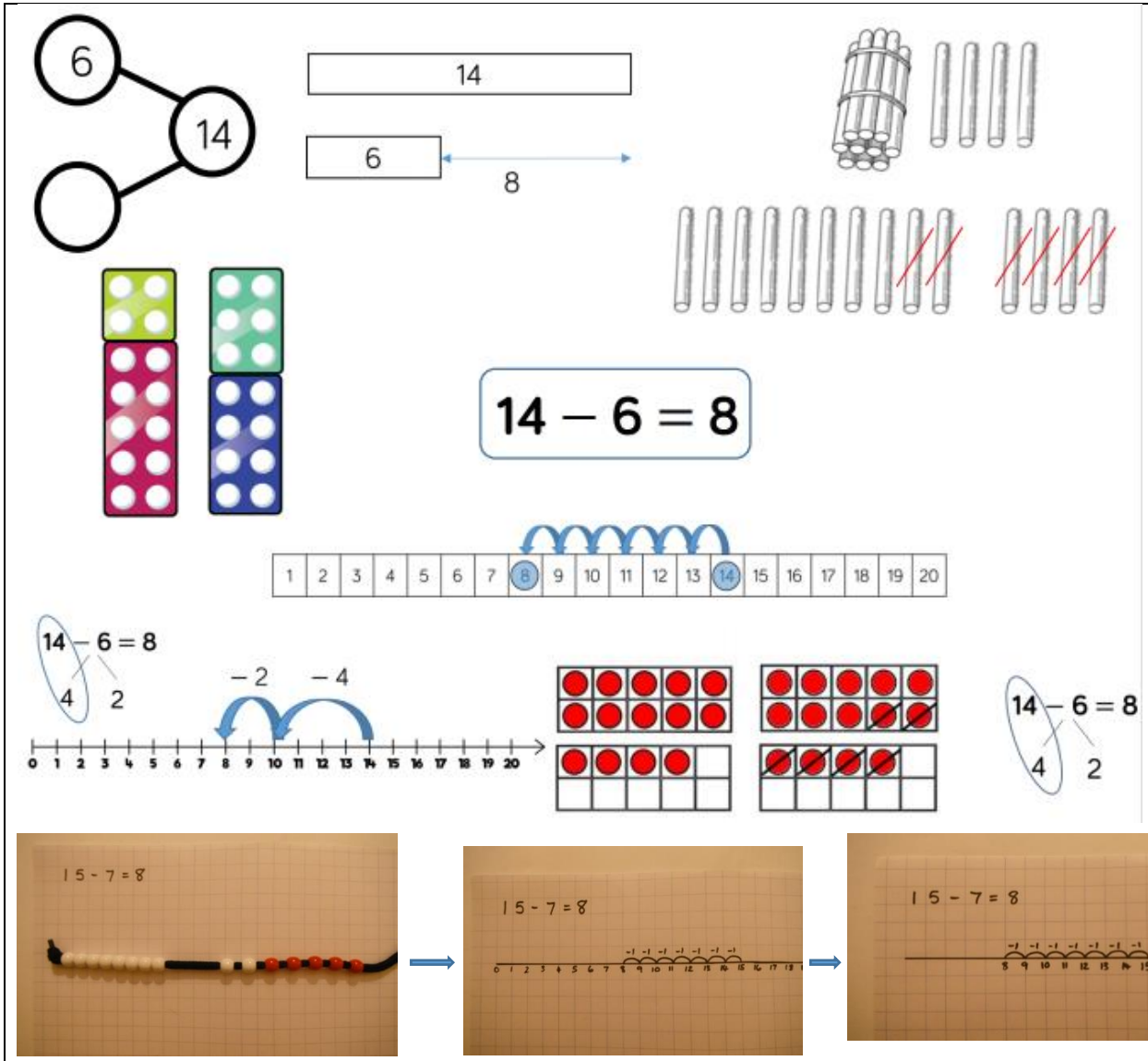
- Bar models (with two bars)
- Cubes

Always write the abstract calculation next to any resources or pictorial representations to support conceptual understanding.

Stage 2: Year 1/2 objectives

Skill: Subtract 1 and 2-digit numbers to 20

Crossing the tens boundary:



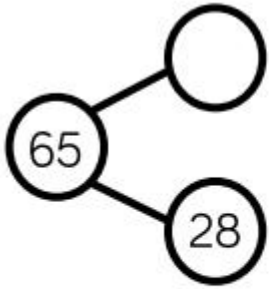
- Highlight the importance of ten ones equaling one ten

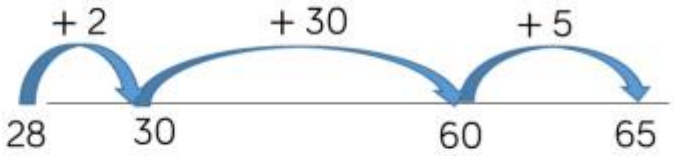
Encourage children to find the number bond to 10 when portioning the subtracted number.

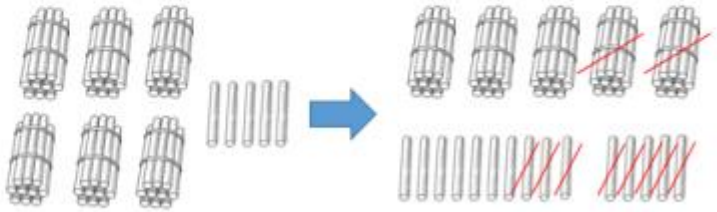
- Numicon, ten frames and number lines support this step well

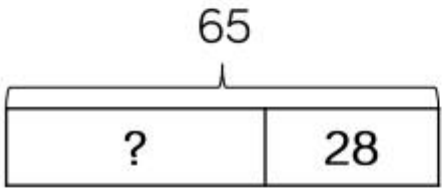
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 3: Year 2 objective

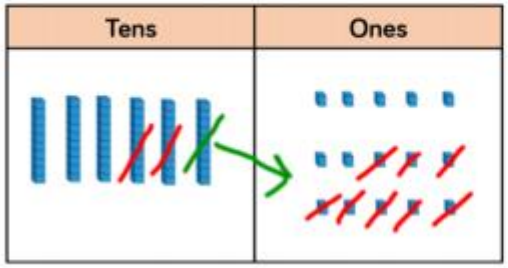




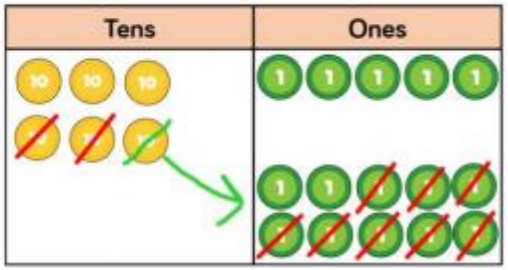


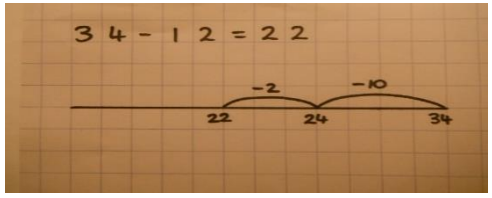


$$65 - 28 = 37$$



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$$





Skill: Subtract 1 and 2-digit numbers to 100

Children should be writing the formal column method alongside their use of concrete resources.

- Straws become less efficient with larger numbers

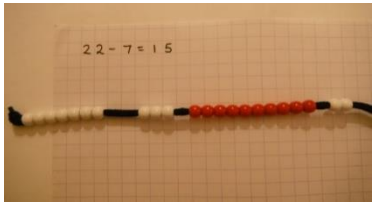
Blank number lines are useful when children are counting on to find the difference.

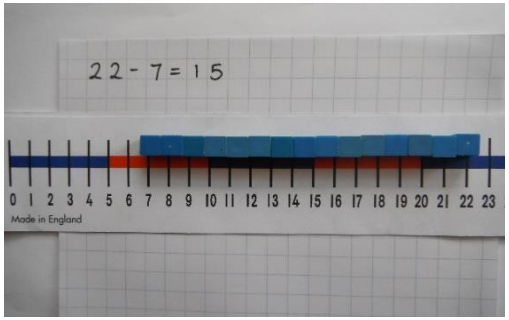
- To be even more efficient, prompt children to jump in multiples of 10.

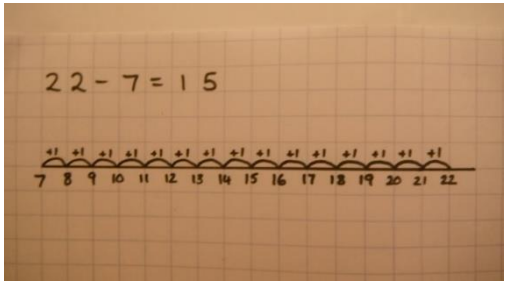
Always model the concrete resources and/or pictorial representations alongside the column method.

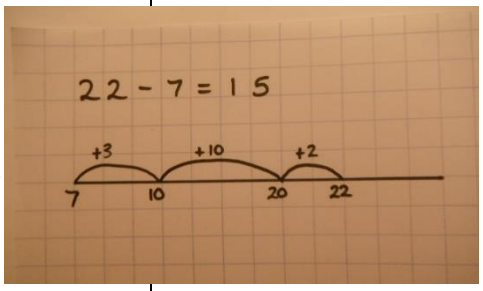
Ensure children do the same in order for them to see how the manipulatives link to the formal method.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

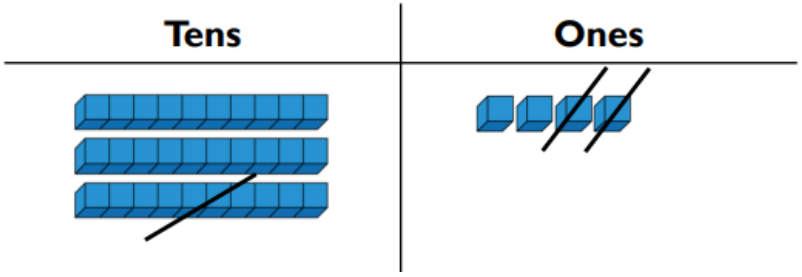








Model



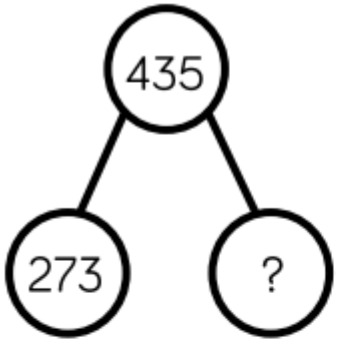
Calculations

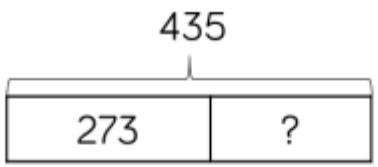
$$\begin{array}{r} 34 \\ -\ 12 \\ \hline 22 \end{array}$$

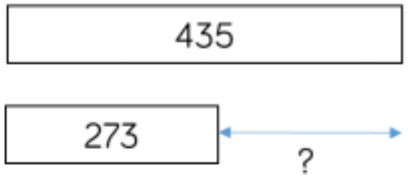
Stage 4: Year 3 objective

Skill: Subtract numbers with up to three digits

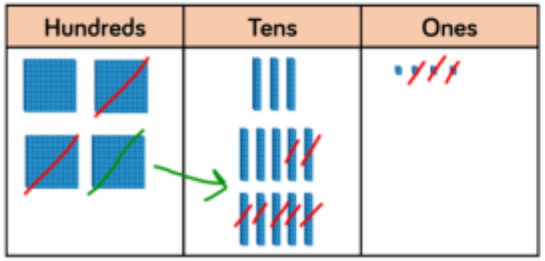
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



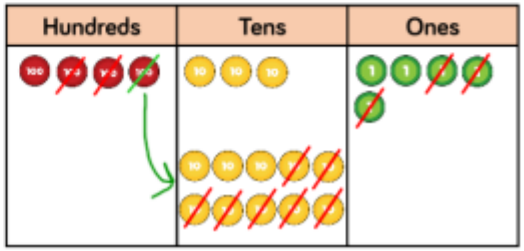




$$435 - 273 = 262$$



$$\begin{array}{r} {}^{3}\cancel{4}^{1}35 \\ -\ 273 \\ \hline 262 \end{array}$$



When subtracting numbers with up to 3 digits, Base 10 and place value counters are the most effective concrete resources.

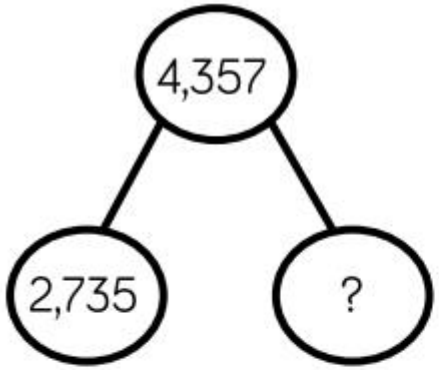
Always model the concrete resources and/or pictorial representations alongside the column method.

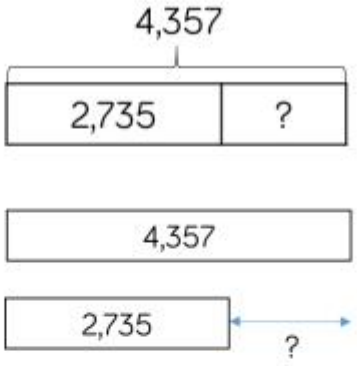
Ensure children do the same in order for them to see how the manipulatives link to the formal method.

Stage 5: Year 4 objective

Skill: Subtract numbers with up to 4 digits

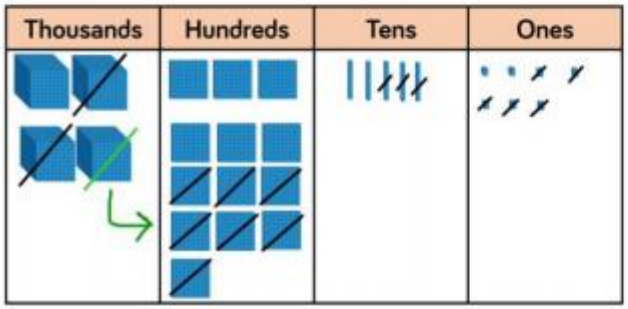
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

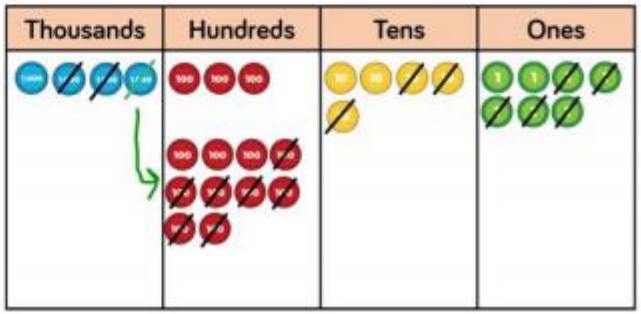




$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

$$4{,}357 - 2{,}735 = 1{,}622$$





When subtracting numbers with up to 4 digits, Base 10 and place value counters are the most effective concrete resources.

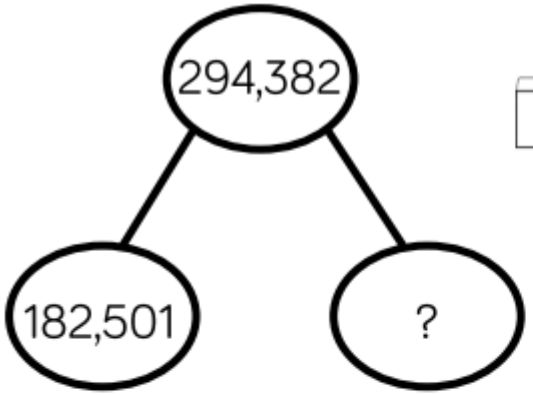
Always model the concrete resources and/or pictorial representations alongside the column method.

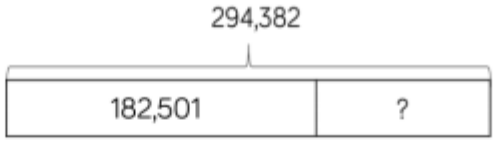
Ensure children do the same in order for them to see how the manipulatives link to the formal method.

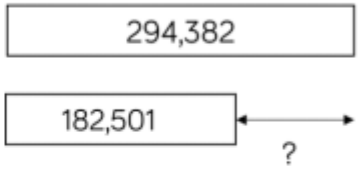
Stage 6: Year 5/6 objectives

Skill: Subtract numbers with more than 4 digits

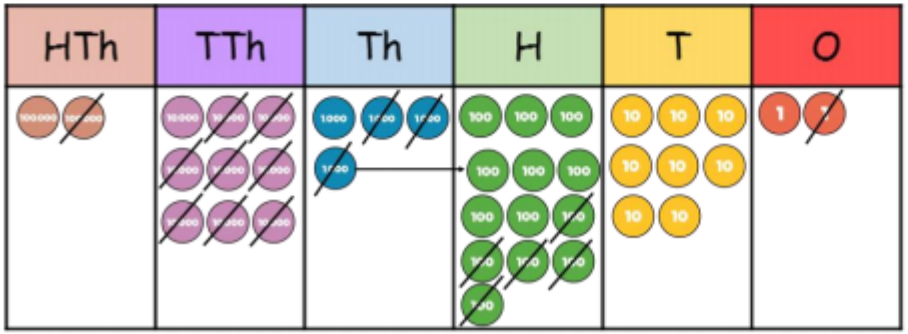
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy







$$294{,}382 - 182{,}501 = 111{,}881$$



	2	9	$\cancel{3}^{2}$	$^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

When subtracting numbers with more than 4 digits, place value counters and plain counters are the most effective concrete resources.

Always model the concrete resources and/or pictorial representations alongside the column method.

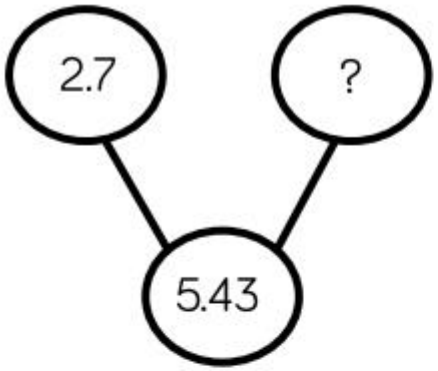
Ensure children do the same in order for them to see how the manipulatives link to the formal method.

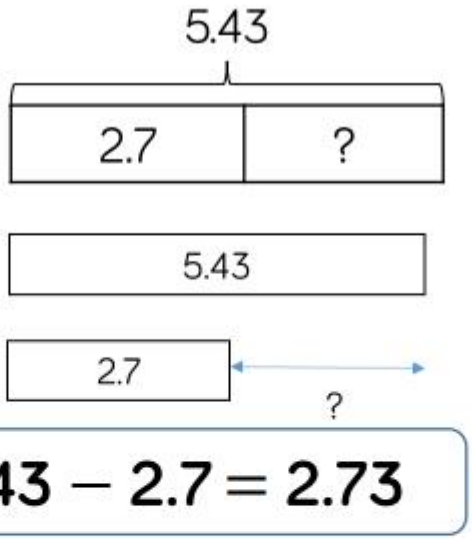
Encourage children to work in the abstract form by using the column method to efficiently add large numbers.

Stage 7: Year 5/6 objectives

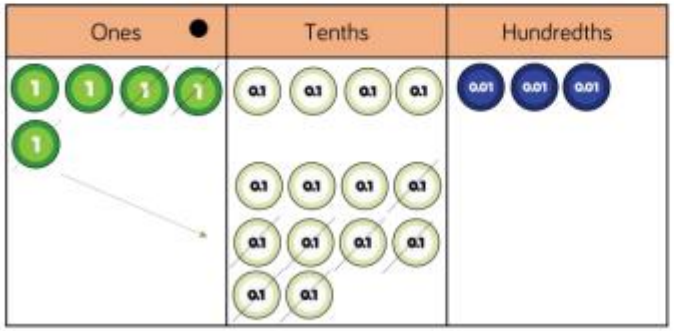
Skill: Subtract decimals with up to 3 decimal places

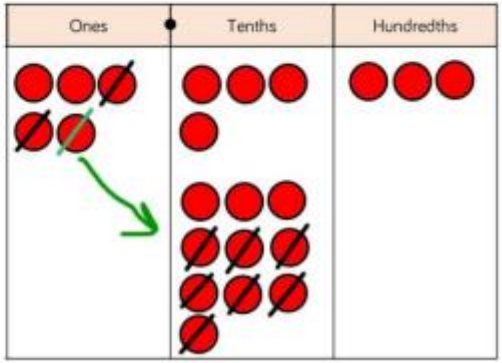
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy





$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$





When subtracting decimals with up to 3 decimal places, place value counters or plain counters on a place value grid are the most effective concrete resources.

Always model the concrete resources and/or pictorial representations alongside the column method. Ensure children do the same in order for them to see how the manipulatives link to the formal method.

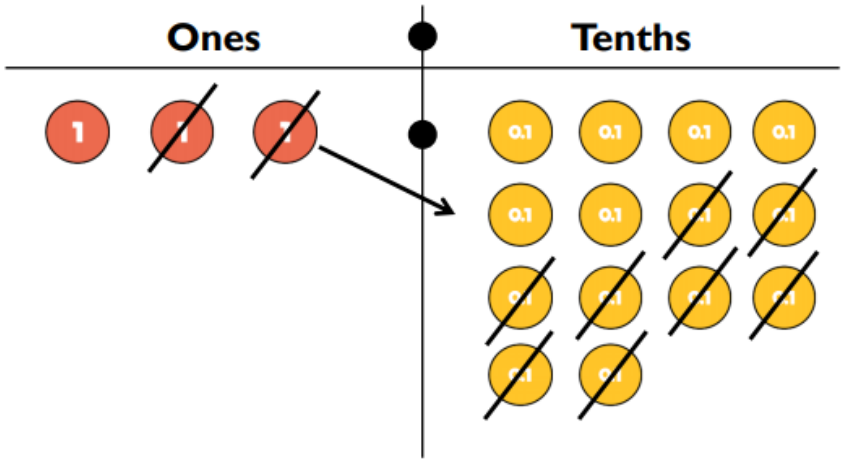
Encourage children to work in the abstract form by using the column method to efficiently add large numbers.

Adding money should be used at this stage to add context.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

$$3.4 - 1.8 =$$

Model



Calculations

$$\begin{array}{r} {}^{2}\cancel{3}\,.\,{}^{1}4 \\ -\,1\,.\,8 \\ \hline 1\,.\,6 \\ \hline \end{array}$$

The written method matches the concrete representation.

If there is nothing in a place value column, we use 0 as a place holder.

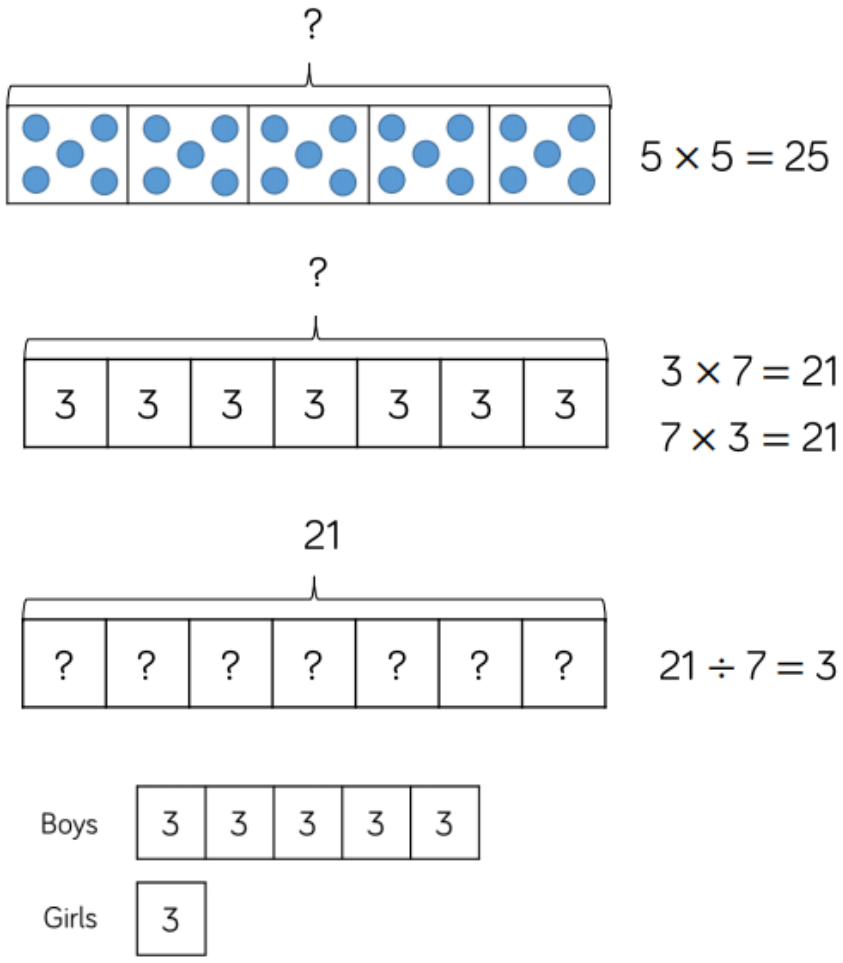
Start from the right hand column and work to the left.

Shows where we can make an exchange.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Overview of the different models – multiplication and division

Bar Model



Key points

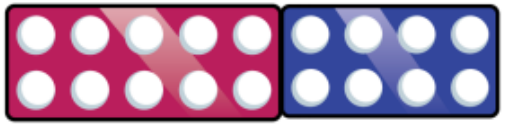
- The single bar model can be used to represent repeated addition.
- Counters, cubes or dots can be placed/drawn within the bar model before moving onto digits.
- Division can be represented by showing the whole written above or in the top bar and then dividing the below bar into equal groups.
- Bar models are a great way for children to draw out and then represent what the knowns and unknowns are within a problem.
- Children then use an appropriate operation to solve the calculation.
- When working out scaling problems, more than one bar model is useful to represent this type of problem.
- For example, there are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
- Multiple bar models provide an opportunity to compare the groups.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

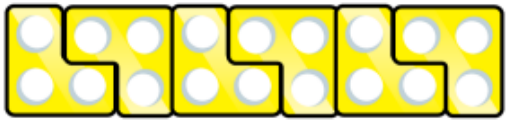
Numicon

$5 \times 4 = 20$
$4 \times 5 = 20$

$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Key points

- Numicon can be used to represent repeated addition.
- Multiplications can be built in a row by placing down the numicon.
- Can should interlock the shapes when using odd numbers.
- They can check the total by placing other numicon pieces on top, such as the tens pieces.
- The following patterns within multiplication can be seen when using numicon:
- Odd x odd = even
- Odd x even = odd
- Even x even = even
- Numicon can be used to support children's understanding of grouping when dividing.
- Children can make the number they are dividing and then place the number they are diving by over the top to find how many groups of the number there are.
- For example, there are 6 groups of 3 in 18.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Bead Strings	**Key points**
 $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 3 = 5$  $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 5 = 3$  $4 \times 5 = 20$ $5 \times 4 = 20$ $20 \div 4 = 5$	• Can support children in their understanding of multiplication as repeated addition. • Encourage children to count in multiples as they build the number. • Children can use the bead string to count forwards and backwards in multiples as they move the beads. • Children can build the number they are dividing and then group the beads into the number they are dividing by. • For example, 20 divided by 4. The children make 20 first. Then they group the beads into groups of 4. Finally they count how many groups they have made to find the answer.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number tracks

$6 \times 3 = 18$
$3 \times 6 = 18$

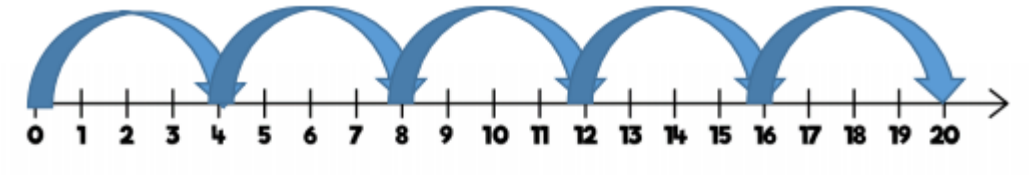
$18 \div 3 = 6$

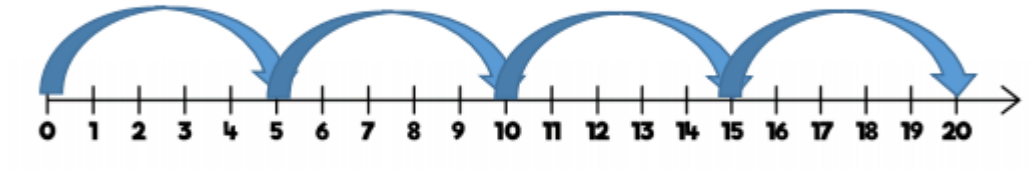
Key points

- Support children's counting in multiples, forwards and backwards.
- Children can keep track of their counting by moving counters or cubes along the track.
- Translucent counters can be used to help children see the number they have landed on.
- Children place their counter on 0 to start and then count on to find the product, when multiplying.
- When dividing, children place thei counter of the number they are diving and then count back in jumps of the number they are dividing by until they reach 0.
- The number of jumps made provides children with the answer.
- Become less efficient with larger numbers.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number lines (labelled)





$4 \times 5 = 20$

$5 \times 4 = 20$

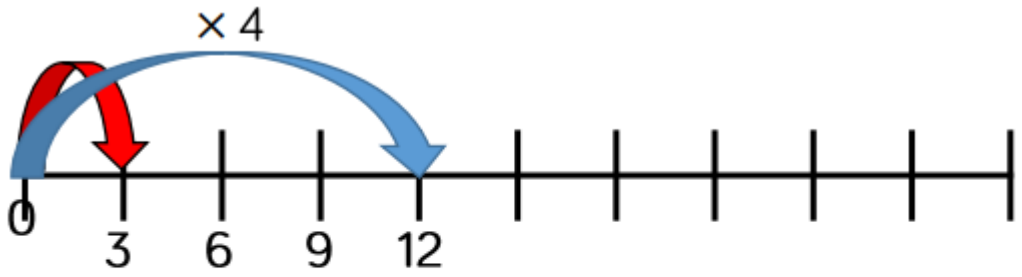
$20 \div 4 = 5$

Key points

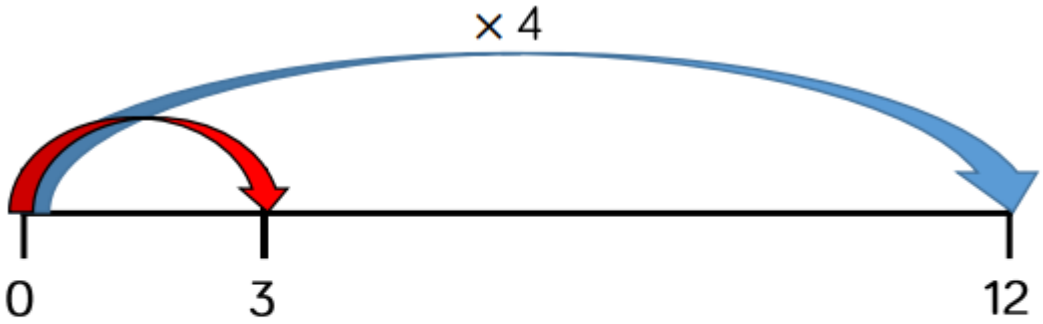
- Can be used for counting in multiples, forwards and backwards as well as calculating single-digit multiplications.
- Children start at 0 and count on to find the product of numbers when multiplying.
- When dividing, children start at the number they are diving and count back in jumps of the umber they are dividing by until they reach 0.
- The number of jumps made provides children with the answer to the division.
- Become inefficient as numbers become larger.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Number lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



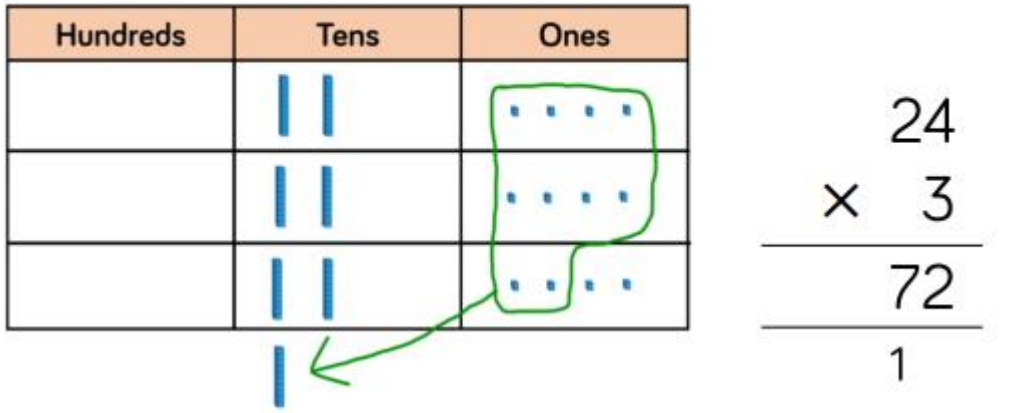
A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Key points

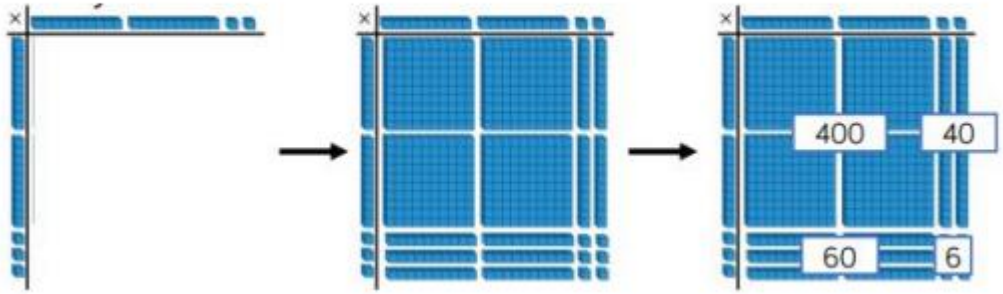
- Can be used to represent scaling as multiplication or division.
- Black number lines with intervals can be used to support children when representing scaling accurately.
- Children can calculate scaling problems by labelling intervals with multiples.
- Children can also use blank number lines without intervals to represent scaling.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$

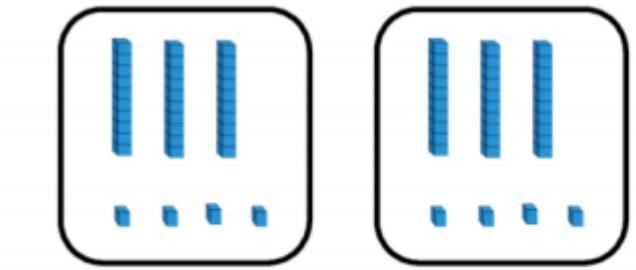


Key points

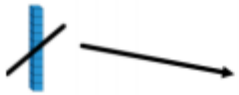
- Provides an effective way to support children's understanding of column multiplication.
- Children need to write out the formal method when working with concrete resources or pictorial representations to help build their conceptual understanding.
- Become less efficient as the numbers or amounts of groups become larger.
- This is due to the amount of equipment needed and the number of exchanges required.
- Provides support for the area model of multiplication.
- Children build the number in a rectangular shape which they then find the area of by calculating the total value of all the pieces.
- The area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Base 10/Dienes (division)

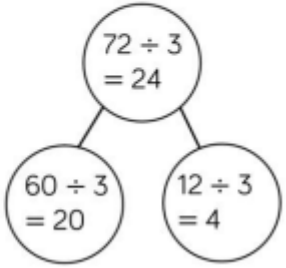


$68 \div 2 = 34$



Tens	Ones
	
	
	

$72 \div 3 = 24$

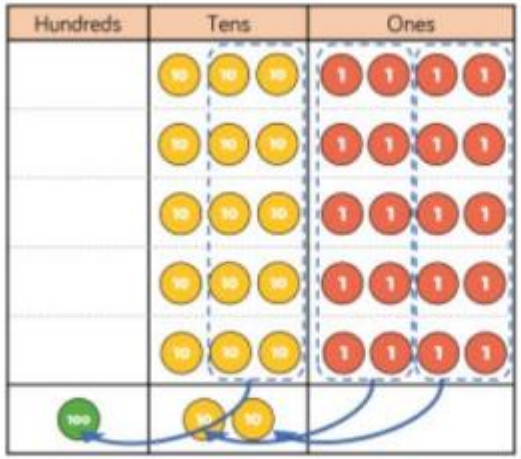


Key points

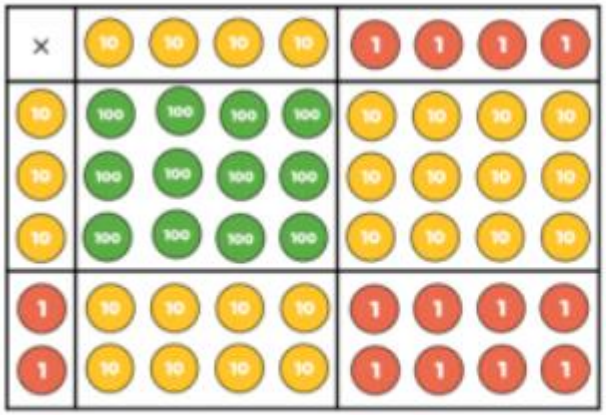
- As numbers become larger, Base 10/Dienes can be an effective way of moving children on from representing numbers as ones to representing them as tens and ones in order to divide.
- Children can share the equipment between different groups by drawing circles or creating rows on a place value grid.
- When sharing, Children start with the larger place value and work from left to right.
- If there are any left in a column, they exchange.
- For example, one ten for ten ones.
- Encourage children to use the part-whole model when recording so they consider how the number has been partitioned in order to divide.
- This will support them with mental methods.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Place value counters (multiplication)



$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 120 \\ \hline 1\ 2 \end{array}$$



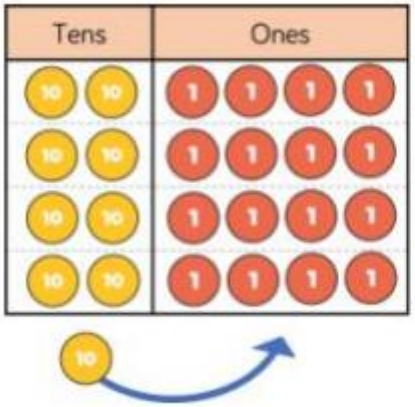
$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

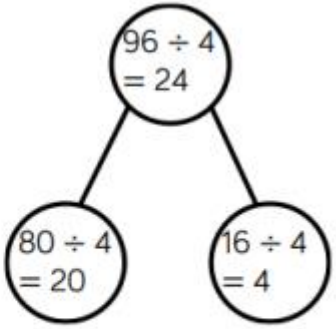
Key points

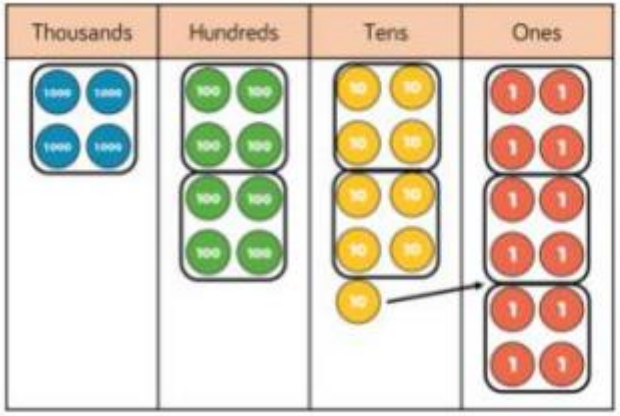
- Provides an effective way to support column multiplication.
- Children will need to write out the formal calculation alongside their resources/pictures to develop their conceptual understanding.
- Counters should replace Base 10/Dienes once the numbers or amount of groups become larger.
- Counters should be used to support the understanding of the written method instead of supporting the arithmetic.
- Place value counters also support the area model of multiplication.
- Children can see how to multiply 2-digit numbers by 2- digit numbers.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Place value counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)4892^{1}} \end{array}$$

Key points

- Children can share place value counters between groups when working with smaller numbers.
- They start by sharing the larger place value column and work from left to right.
- Any counters left over once they have been shared are exchanged.
- Using place value counters also supports children's understanding of short division by grouping the counters instead of sharing them.
- Children work from left to right through the place value columns and group the counters in the number they are dividing by.
- Any counters left over once they have been grouped are exchanged.

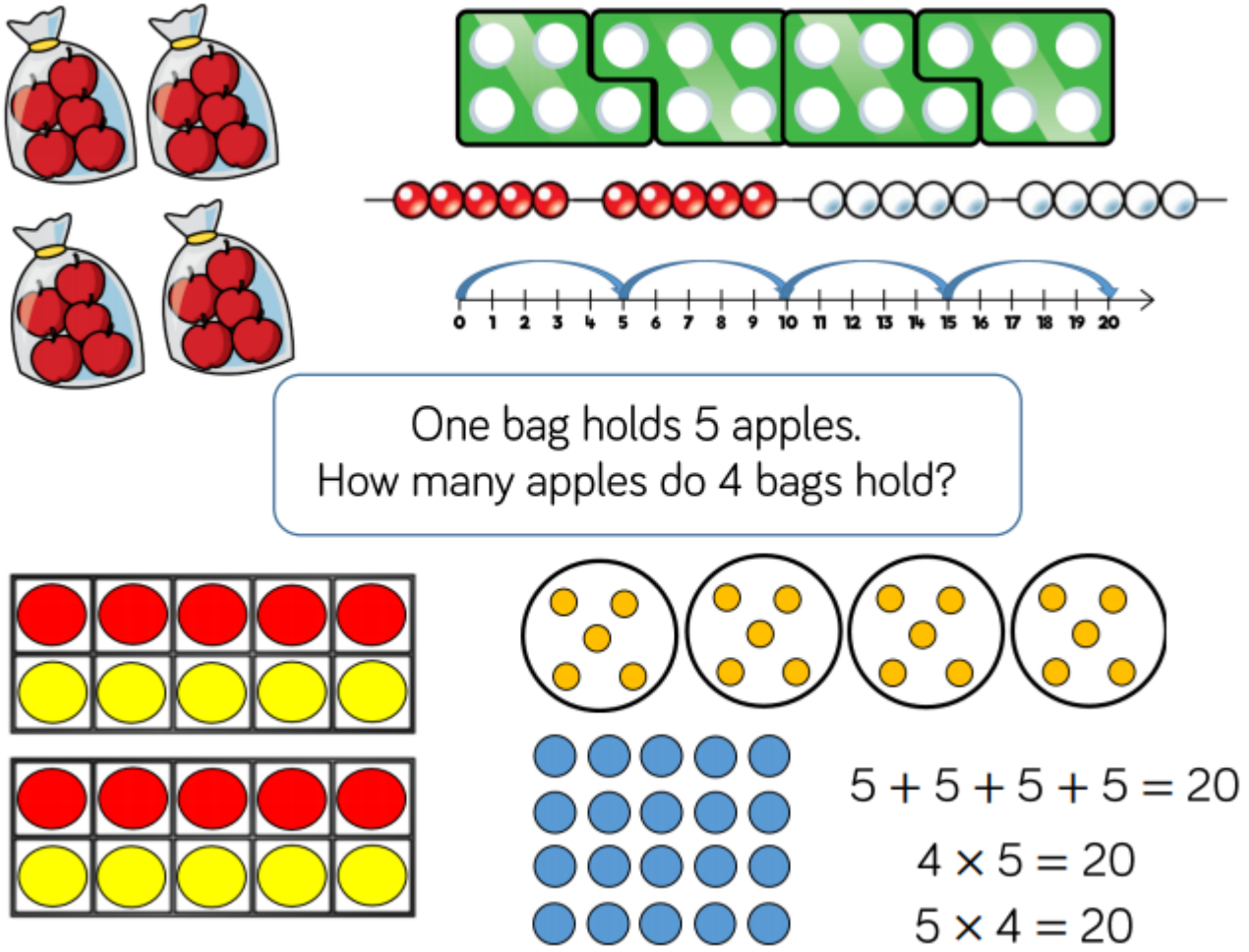
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Multiplication

Key Vocabulary Array – an ordered collection of counters, cubes or other item in rows and columns. Commutative – numbers can be multiplied in any order. Exchange – change a number or expression for another equal value. Multiplicand – a number to be multiplied by another	Partitioning – splitting a number into its component parts. Product – the result of multiplying one number by another. Scaling – enlarging or reducing a number by a given amount, called the scale factor.
Mental Strategies (see *Teaching Children to Calculate Mentally* for more details and activities to develop these) Using known multiplication facts to 12 x 12 Doubling (including multiplying by 4 by doubling twice, multiplying by 8 by doubling 3 times, multiplying by 5 by multiplying by 10 then halving, etc.) Multiplying by multiples of 10 Multiplying by one-digit numbers and two-digit numbers	

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 1: Year 1/2 objectives



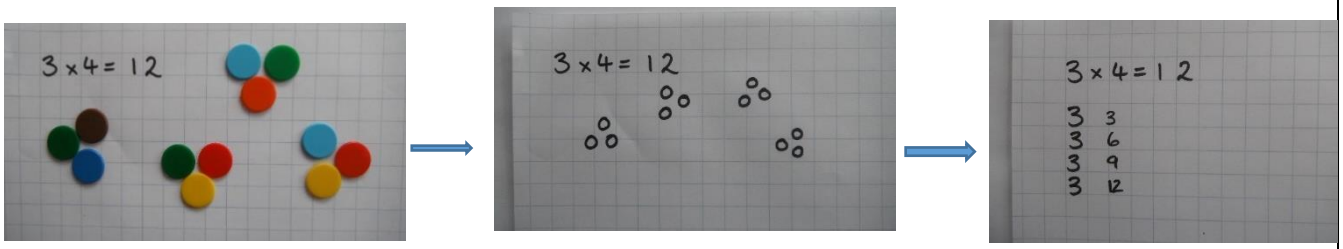
Skill: Solve 1-step problems using multiplication

At this stage, children represent multiplication as repeated addition in lots of different ways.

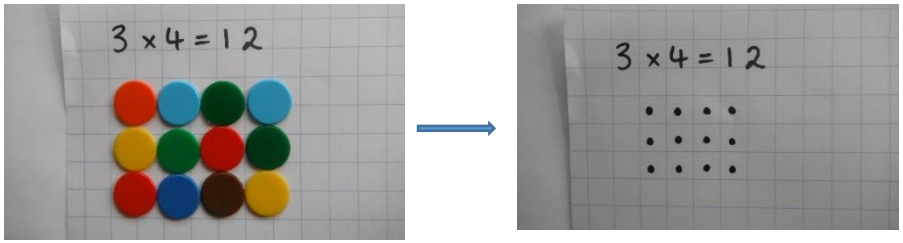
In Year 1, children use concrete and pictorial representations to solve problems.

In Year 2, children are introduced to the multiplication symbol.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



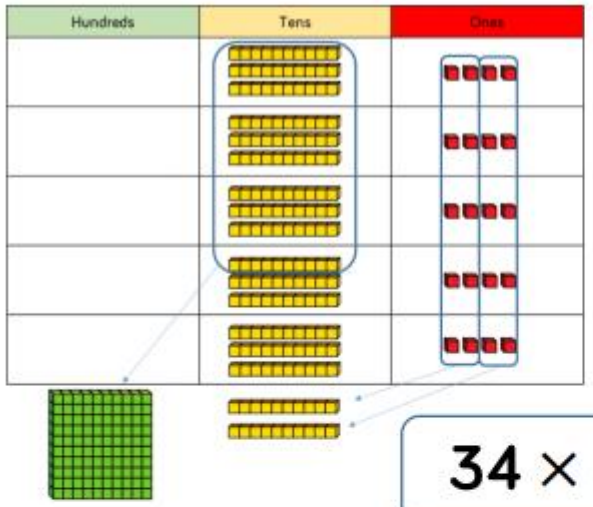
Lots of – grouping



Arrays

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

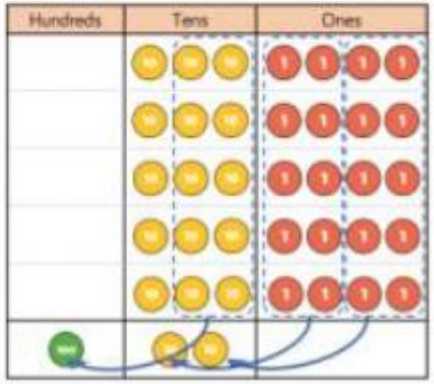
Stage 2: Year 3/4 objectives



	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

$$34 \times 5 = 170$$

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		



Skill: Multiply 2-digit numbers by 1-digit numbers

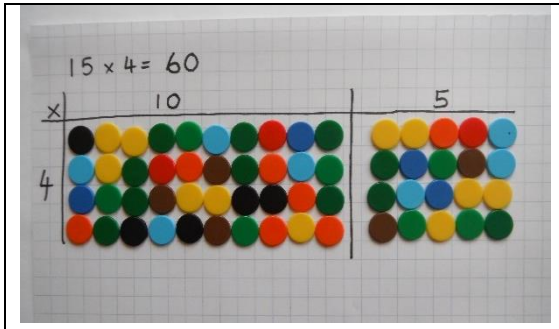
Begin by teaching the expanded column method before moving onto the short multiplication method.

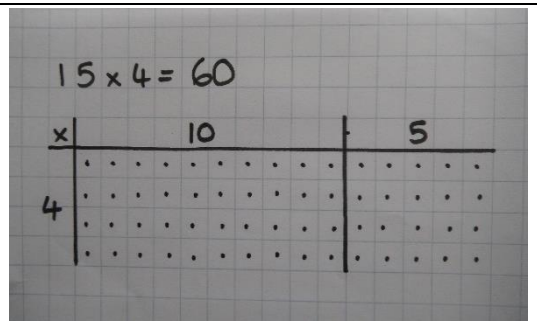
Children should use their times table knowledge when working out individual stages of the calculation.

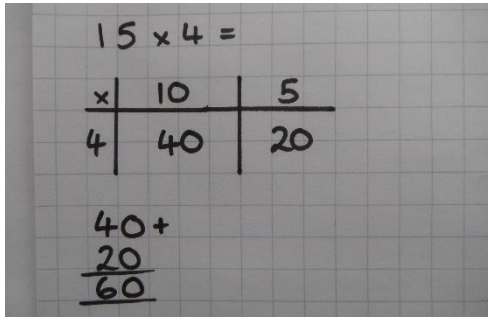
Place value counters should be used to support understanding of the method rather than supporting the multiplication.

Grid method

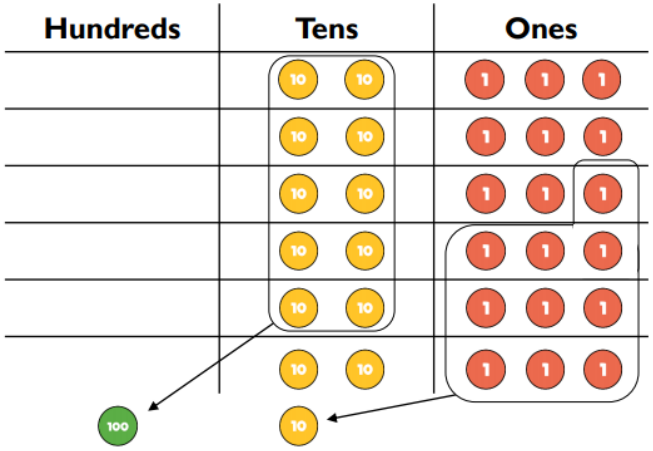
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy







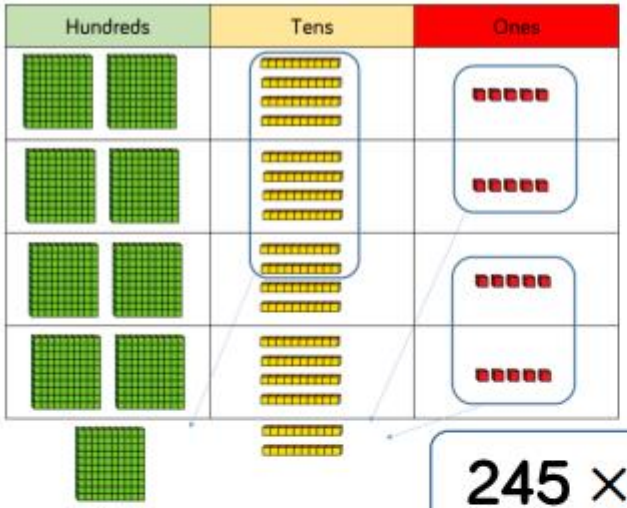
Model

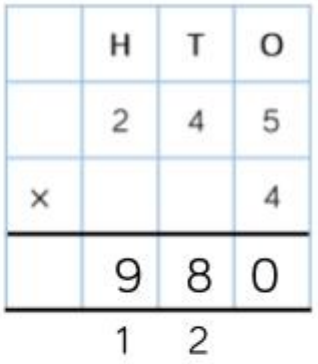


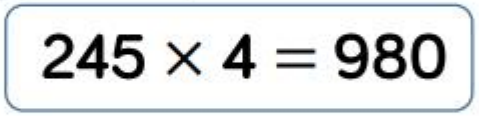
Calculations

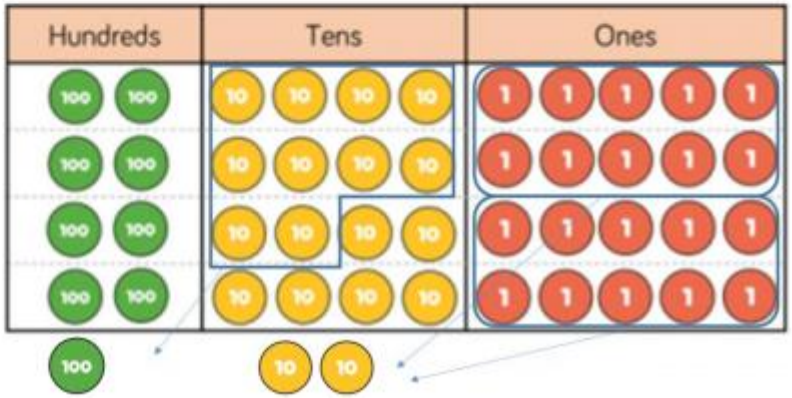
$$\begin{array}{r} 2\ 3 \\ \times\quad 6 \\ \hline 1\ 3\ 8 \\ \hline 1\ 1 \end{array}$$

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

<table>
<tr>
<td>Stage 3: Year 3/4 objectives
 






</td>
<td>Skill: Multiply 3-digit numbers by 1-digit numbers

At this stage, encourage children to move onto the short, formal method of multiplication.

Use Base 10 and place value counters to continue to support children’s understanding of the written method.

Limit the number of exchanges needed and move children away from resources when multiplying larger numbers.</td>
</tr>
<tr>
<td>Stage 4: Year 5 objective</td>
<td>Skill: Multiply 4-digit numbers by 1-digit numbers</td>
</tr>
</table>

$$1{,}826 \times 3 = 5{,}478$$

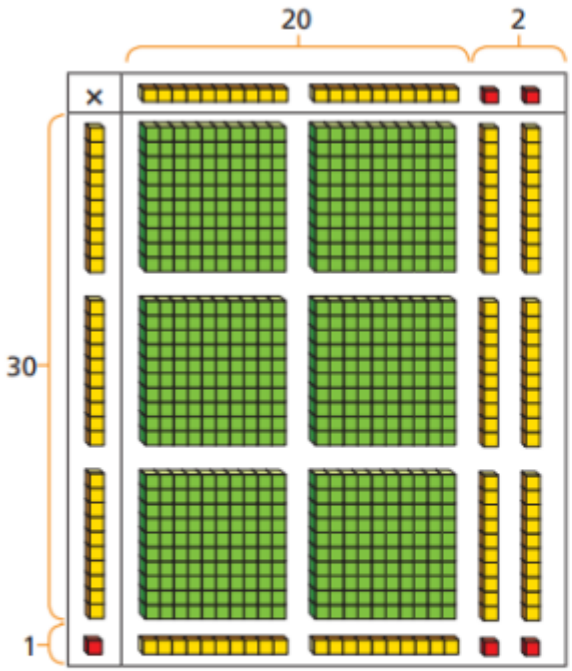
	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

Place value counters are the best manipulative to use when supporting children's understanding of the formal written method.

Allow children who are struggling with their times table knowledge to use multiplication grids so they can focus on the use of the written method.

Stage 5: Year 5 objective

Skill: Multiply 2-digit numbers by 2-digit numbers



×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

Use the area model to help children understand the size of the numbers they are using.

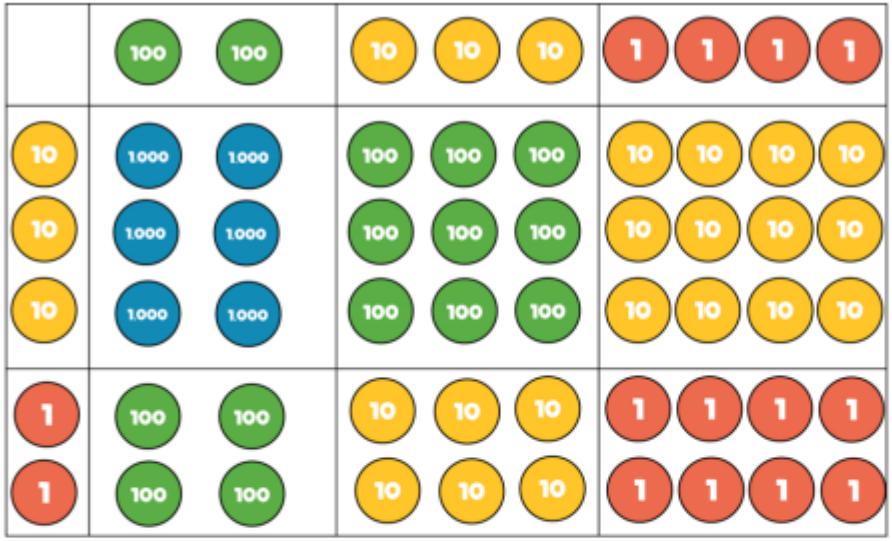
This links to finding the area of a rectangle by finding the space covered by the Base 10.

The grid method matches the area model as an initial written method before moving on to the formal written method.

Stage 6: Year 5 objective

Skill: Multiply 3-digit numbers by 2-digit numbers

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_{1}7$	$_{1}0$	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$$234 \times 32 = 7{,}488$$

The area model can continue to be used.

Place value counters are more efficient at this stage than Base 10. However, Base 10 can still be used to highlight the size of numbers.

Encourage children to make links between the grid method and the formal written method.

Stage 7: Year 5/6 objectives

Skill: Multiply 4-digit numbers by 2-digit numbers

By this stage, it is hoped that children are confident in the written method.

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2
		1		

$2{,}739 \times 28 = 76{,}692$

Once again, provide times tables multiplication grids to support children who are struggling with their times tables so they can focus on the use of the method.

Encourage children to be consistent with their placement of exchanged digits.

Division

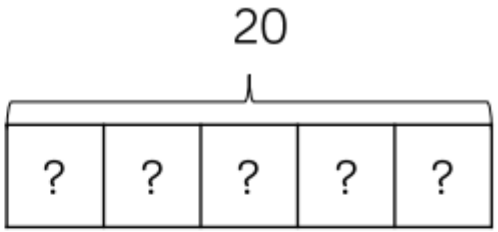
Key Vocabulary	

Dividend – the number that is divided. Divisor – the number by which another is divided Exchange – change a number or expression for another of equal value, Factor – a number that multiples with another to make a product.	Partitioning – splitting a number into its component parts. Quotient – the result of division. Remainder – the amount left over after a division when the divisor is not a factor of the dividend. Scaling – enlarging or reducing a number by a given amount, called the scale factor.

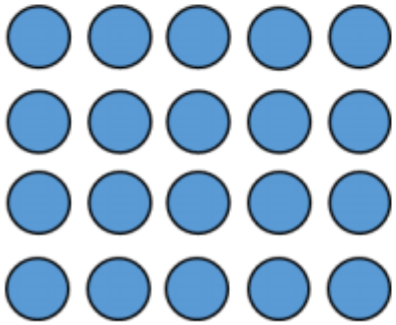
Mental Strategies (see *Teaching Children to Calculate Mentally* for more details and activities to develop these)

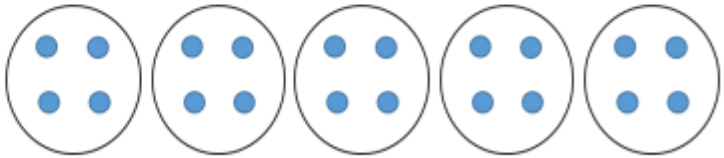
Using known multiplication and division facts to 12 x 12
Halving (including dividing by 4 by halving twice, dividing by 8 by halving 3 times, dividing by 5 by dividing by 10 then doubling, etc.)
Dividing by multiples of 10
Dividing by one-digit numbers and two-digit numbers
Finding fractions, decimals and percentages

Stage 1: Year 1/2 objectives	**Skill: Solve 1-step problems using division (sharing)**



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?





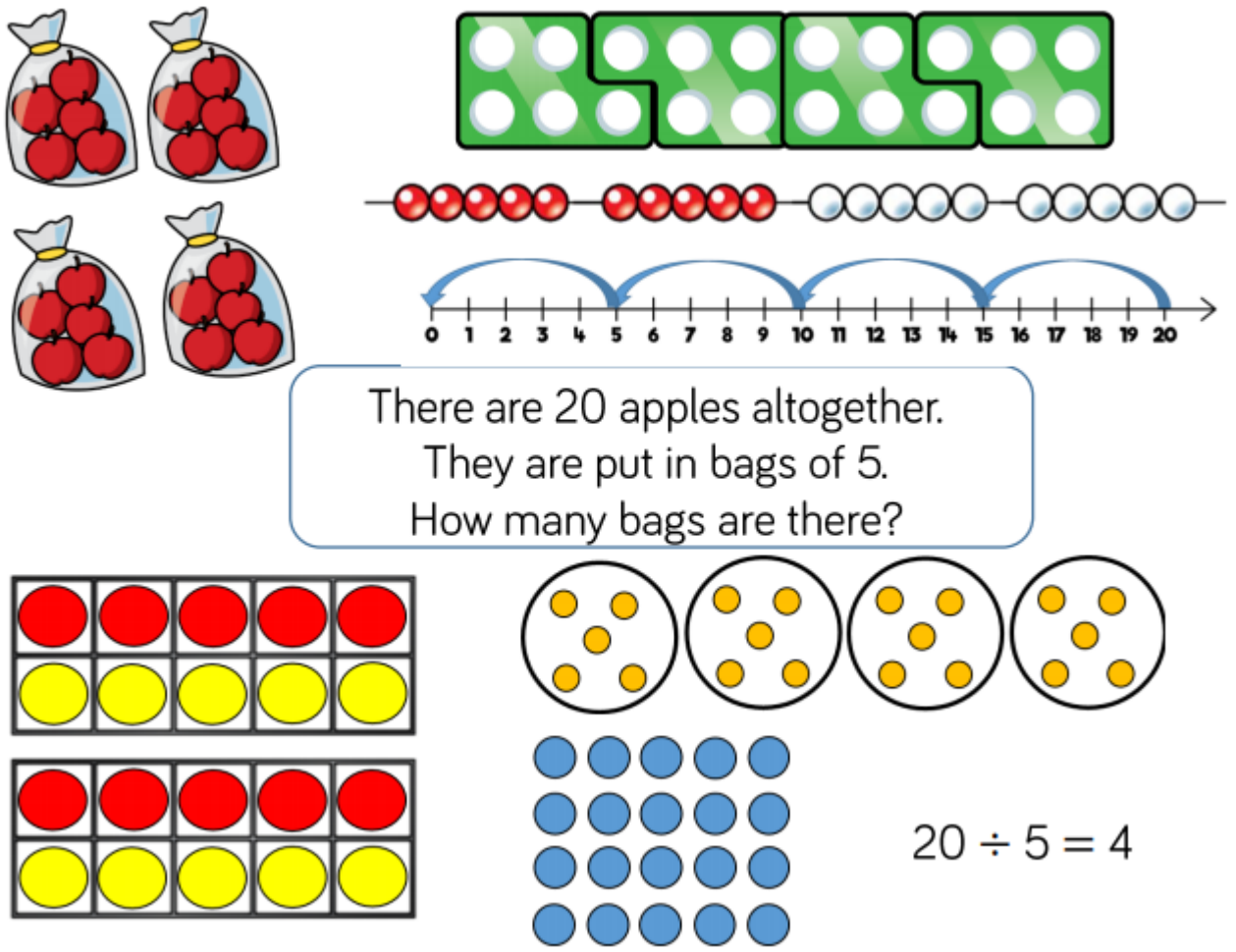
$20 \div 5 = 4$

At this stage, children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems.

In Year 2, children are introduced to the division symbol.

Stage 2: Year 1/2 objectives	**Skill: Solve 1-step problems using division (grouping)**

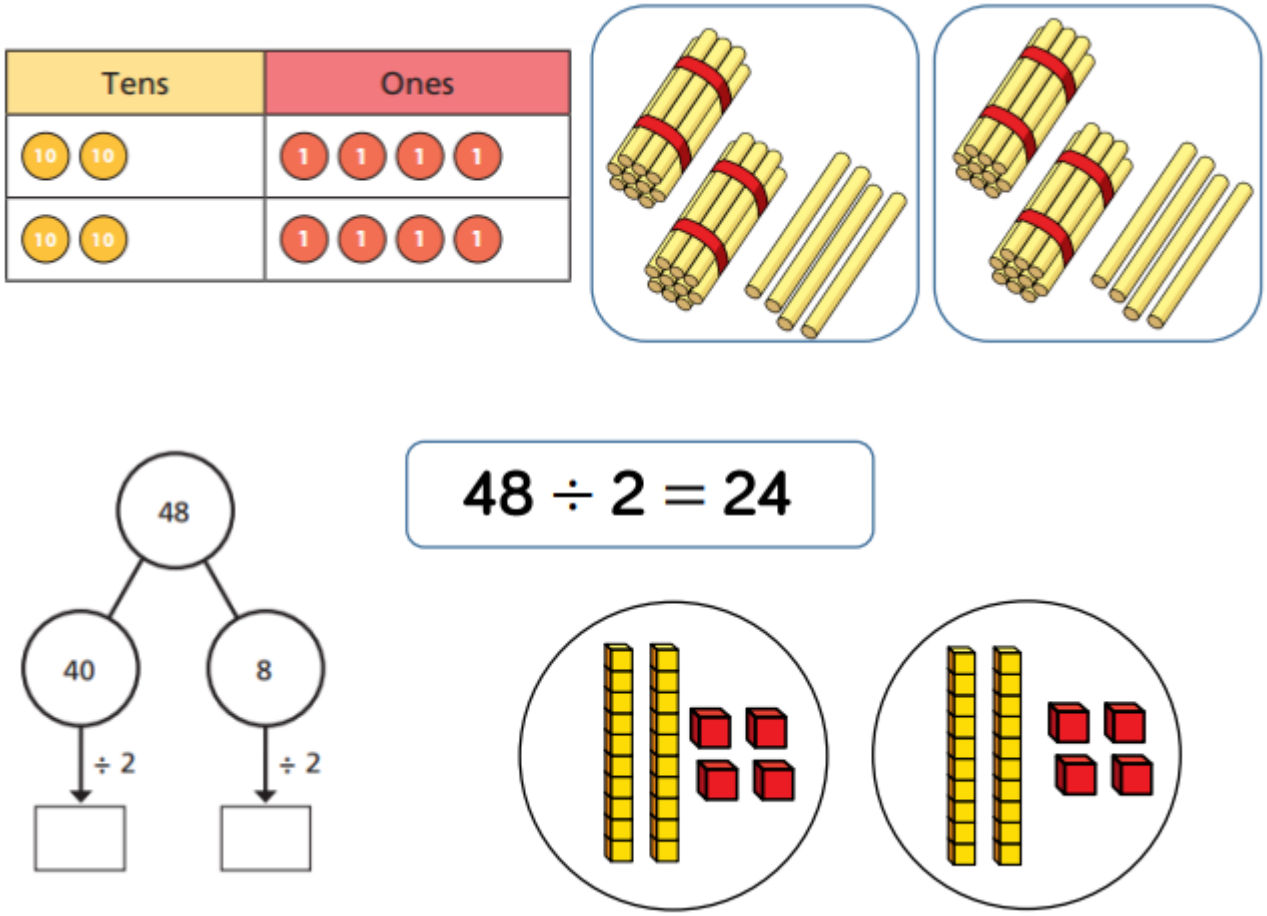


Children solve problems by grouping and counting the number of groups.

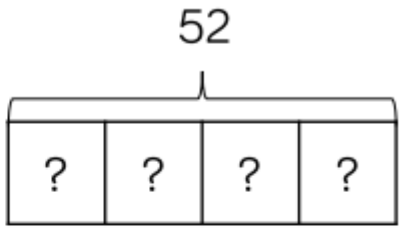
Grouping encourages children to count in multiples and links to repeated subtraction on a number line.

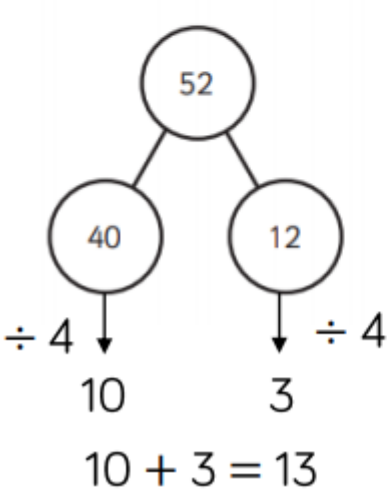
Using fixed groups such as numicon help to show the link between multiplication and division.

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

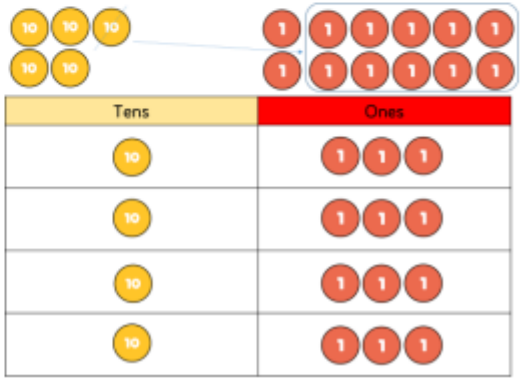
Stage 3: Year 1/2 objectives	**Skill: Divide 2-digit numbers by 1-digit (sharing no exchange)**
	Children can use concrete resources that allow them to partition into tens and ones, when dividing larger numbers. The following manipulatives can be used to share numbers into equal groups: • Straws • Base 10 • Counters Part-whole models provide a written method that matches the concrete representation.
Stage 4: Year 3/4 objectives	**Skill: Divide 2-digits by 1-digit (sharing with exchange)**

Tens	Ones





$$52 \div 4 = 13$$



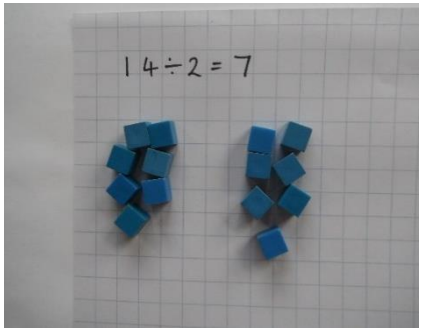
Tens	Ones
10	111
10	111
10	111
10	111

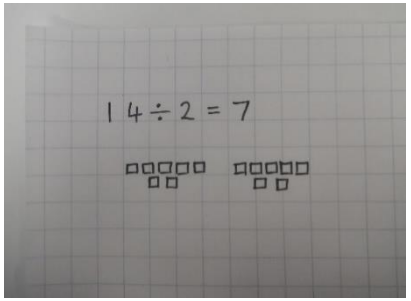
Children can use Base 10 and place value counters to exchange one ten for ten ones.

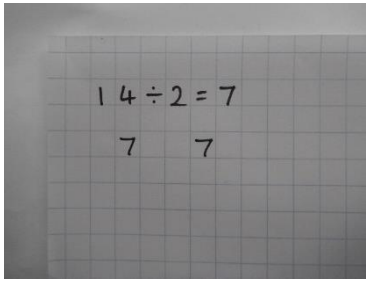
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

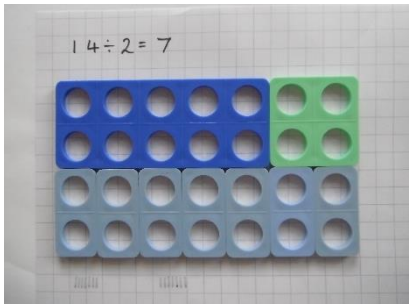
Flexible partitioning in a part-whole model supports this method.

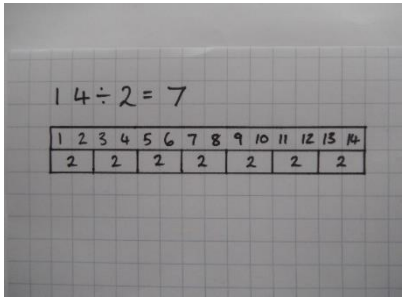
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

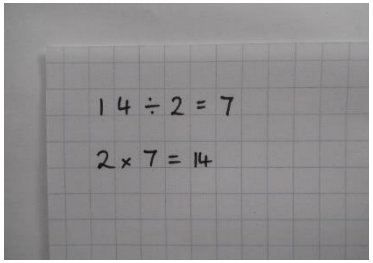


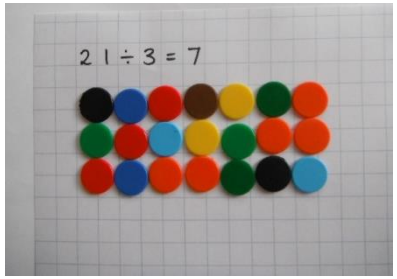


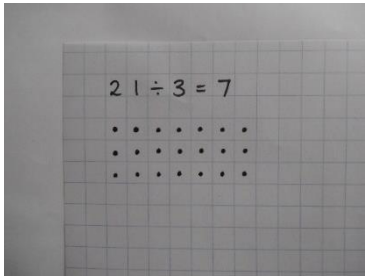


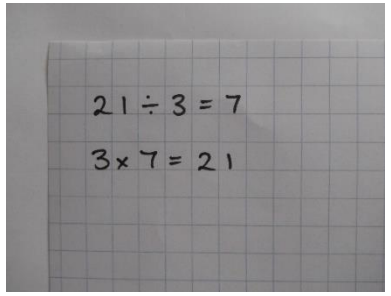




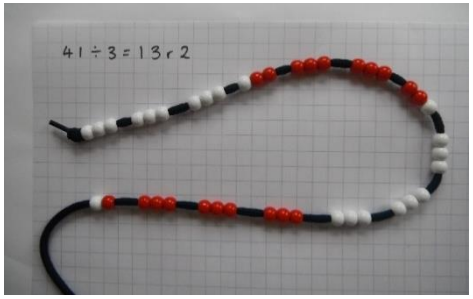


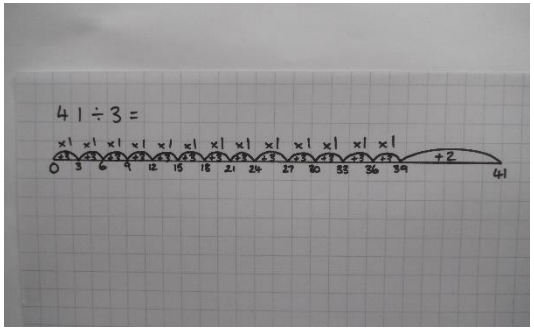


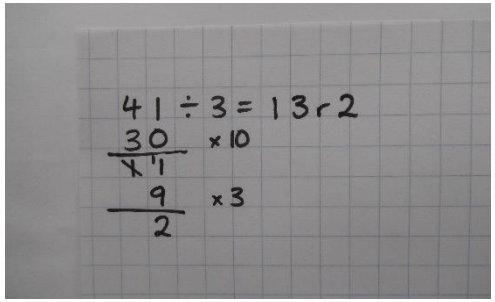


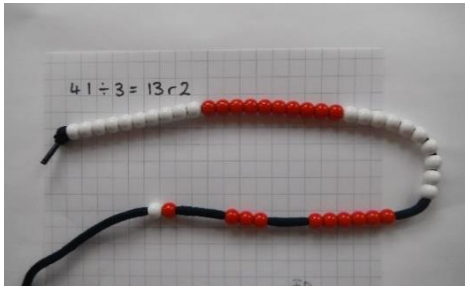


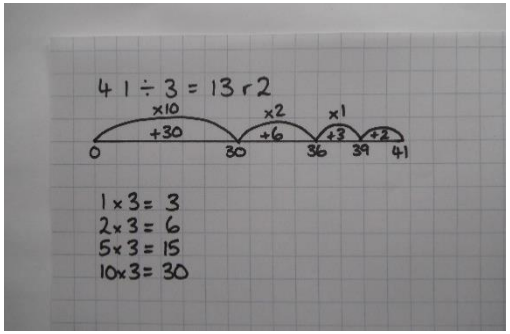
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy





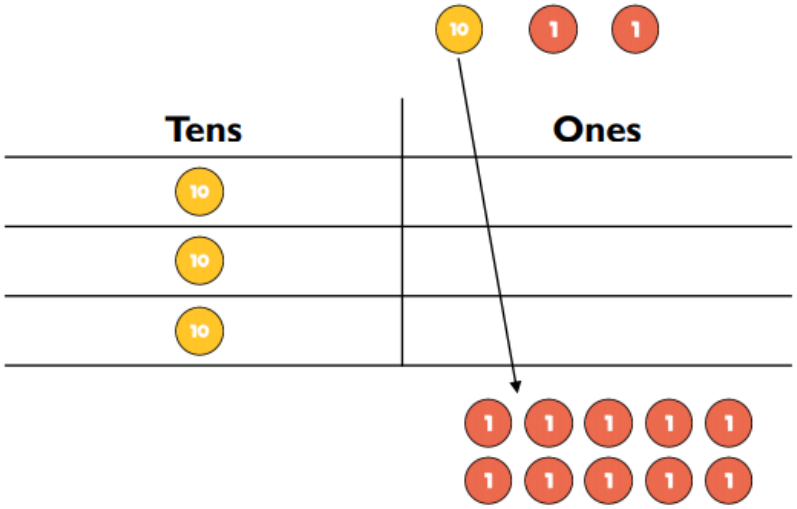






Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

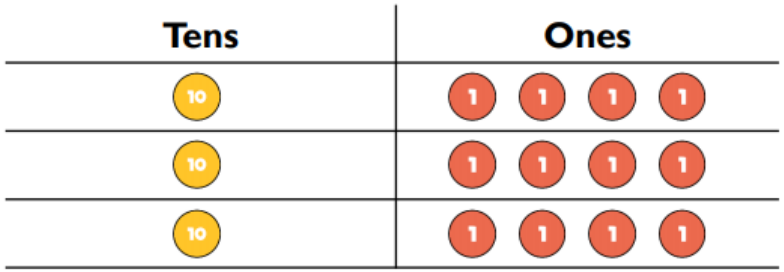
Model



Calculations

$$42 \div 3 =$$

Model



Calculations

$$42 \div 3 = 14$$

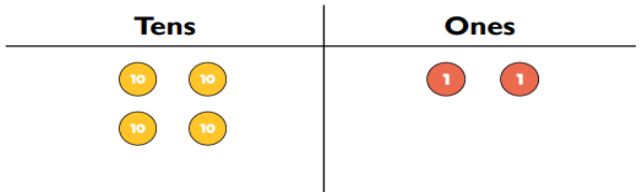
Sharing

There are 42 crayons in a box. They are shared equally between three children. How many crayons do they get each?

Grouping

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

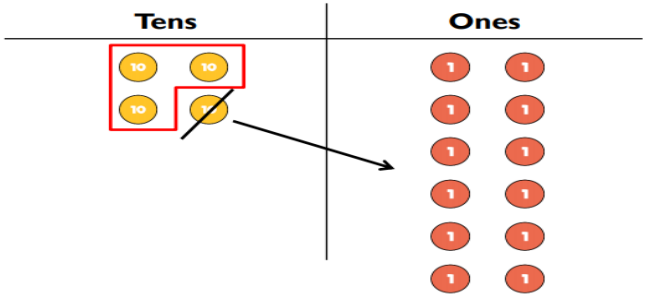
Model



Calculations

$$3 \overline{)\ 4\ \ 2}$$

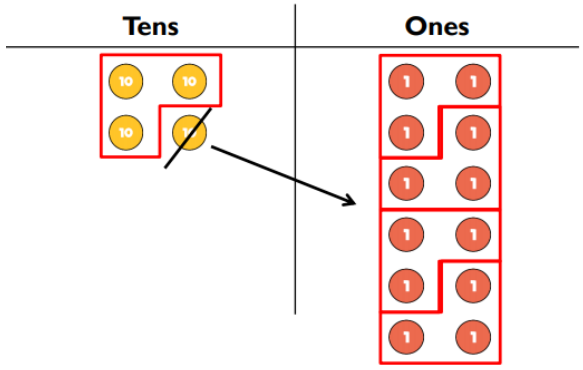
Model



Calculations

$$\begin{array}{r} 1 \\ 3 \overline{)\ 4\ {}^{1}2} \end{array}$$

Model



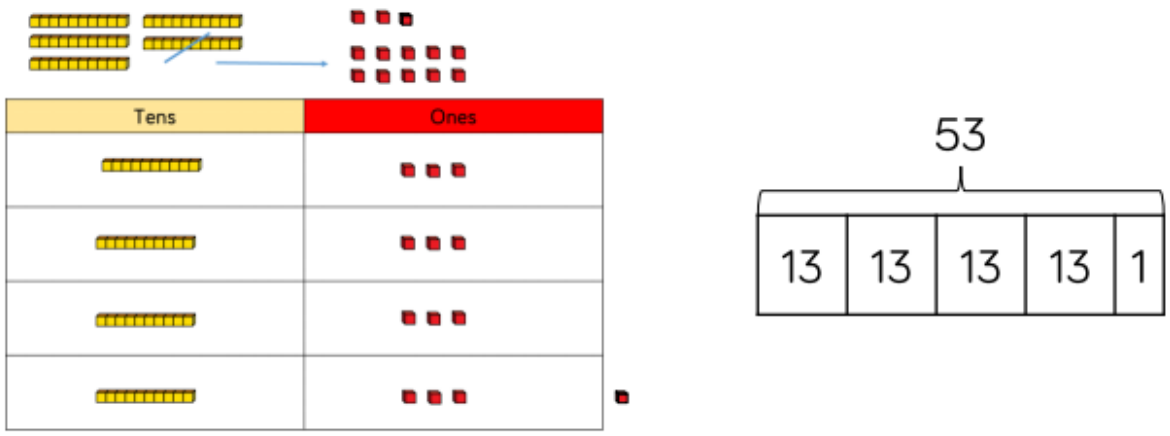
Calculations

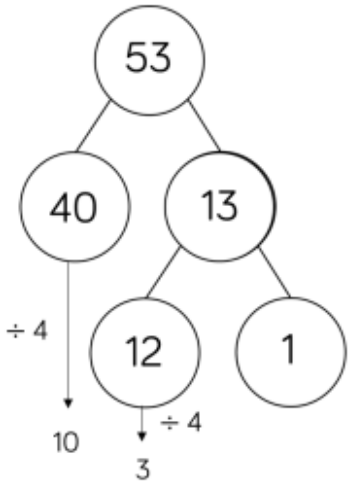
$$\begin{array}{r} 1\ \ 4 \\ 3 \overline{)\ 4\ {}^{1}2} \end{array}$$

There are 42 crayons in a box.
They are put in pots with three in each pot.
How many pots are needed?

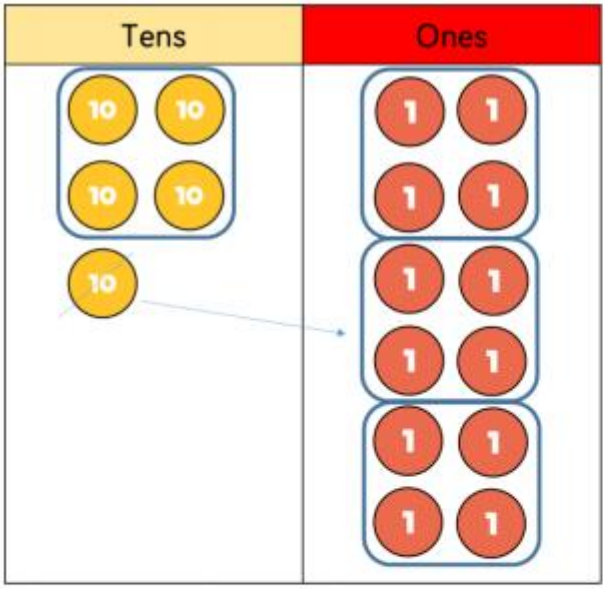
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

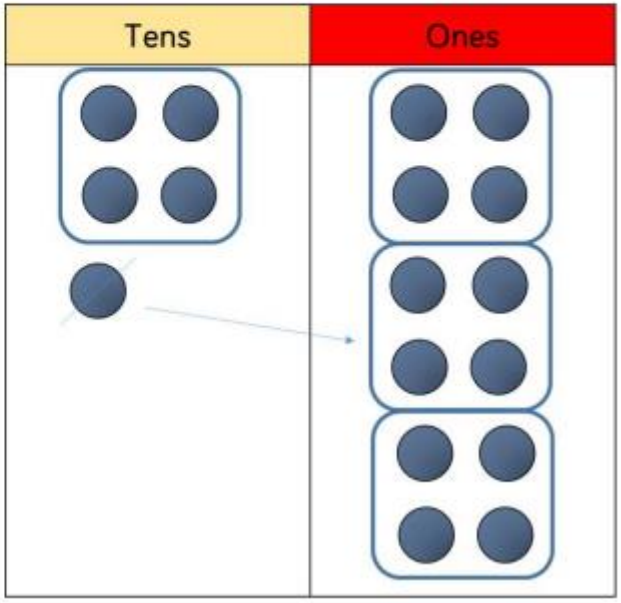
Stage 5: Year 3/4 objectives	**Skill: Divide 2-digit numbers by 1-digit (sharing with remainders)** Children can use Base 10 and place value counters to exchange one ten for ten ones, when dividing numbers with remainders. Starting with the equipment outside of the place value grid will highlight remainders because they will be left outside of the grid once the equal groups have been formed. Flexible partitioning in a part-whole model supports this method.
Stage 6: Year 4/5 objectives	**Skill: Divide 2-digits by 1-digit (grouping)**





$$53 \div 4 = 13\ r1$$





$52 \div 4 = 13$

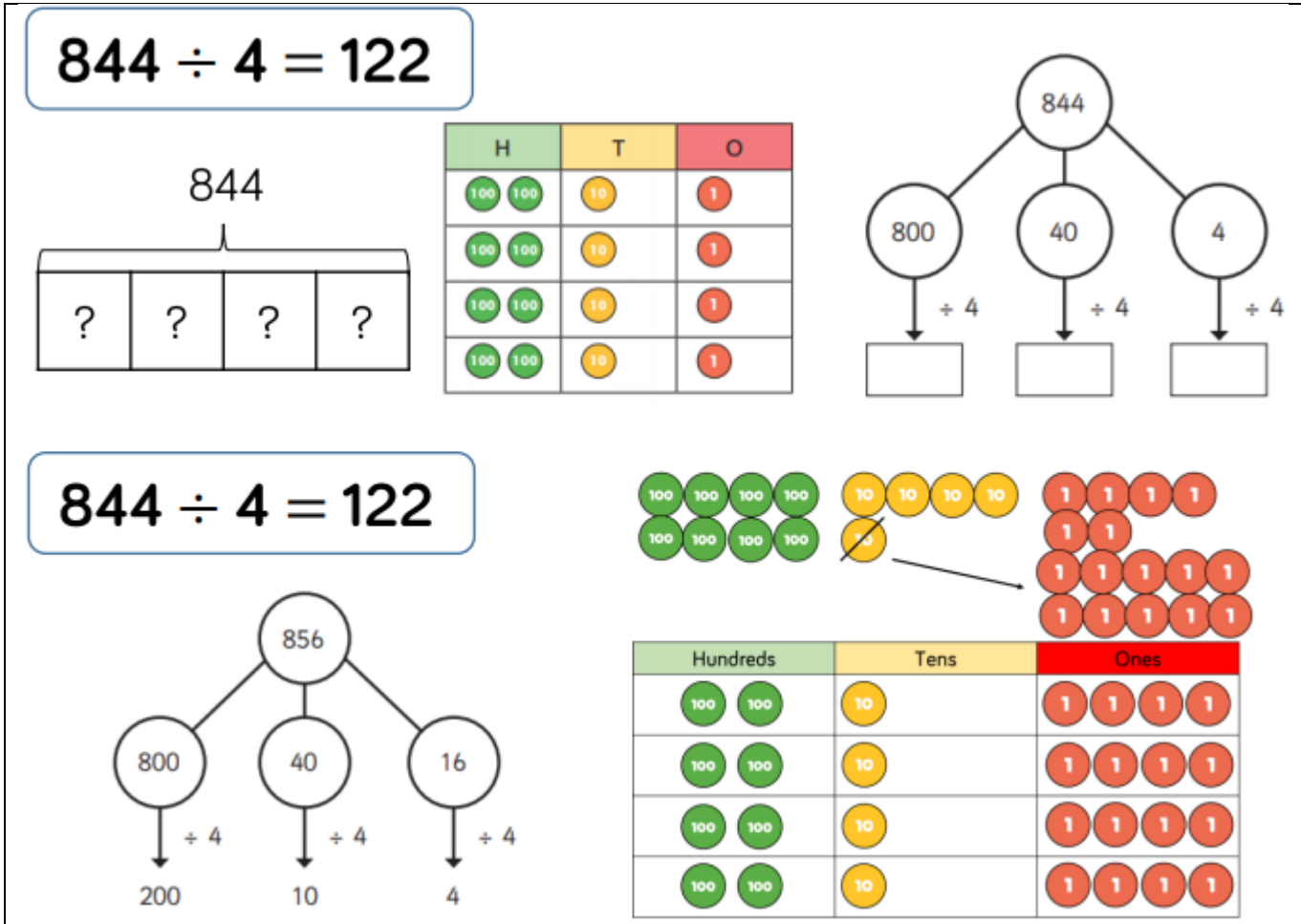
Children use grouping when they divide using the short division method.

Starting with the largest place value, they group by the divisor (in this example, the number 4).

Using the correct vocabulary is important at this stage. Children should consider, ‘How many groups of 4 tens can we make?’ and, ‘How many groups of 4 ones can we make?’

Remainders are highlighted because they are left ungrouped.

Stage 7: Year 4 objective	**Skill: Divide 3-digit by 1-digit (sharing)**



Place value counters can continue to be used when sharing 3-digit numbers into equal groups.

Children should start with the equipment outside the grid before sharing the hundreds, tens and ones equally between the rows.

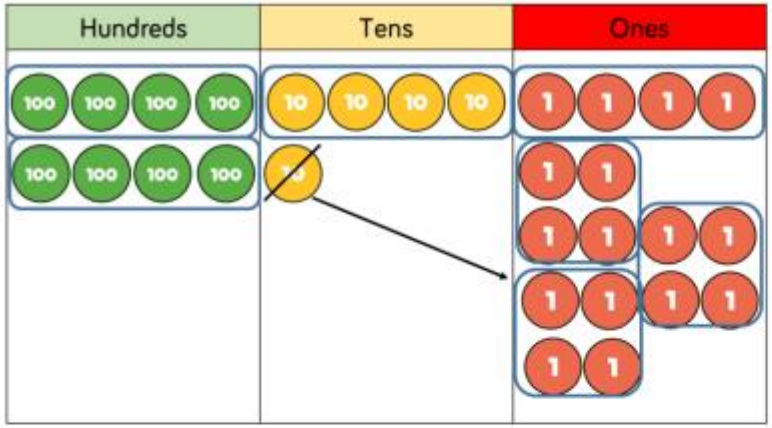
This method also highlights remainders.

Flexible partitioning in a part-whole model supports this method.

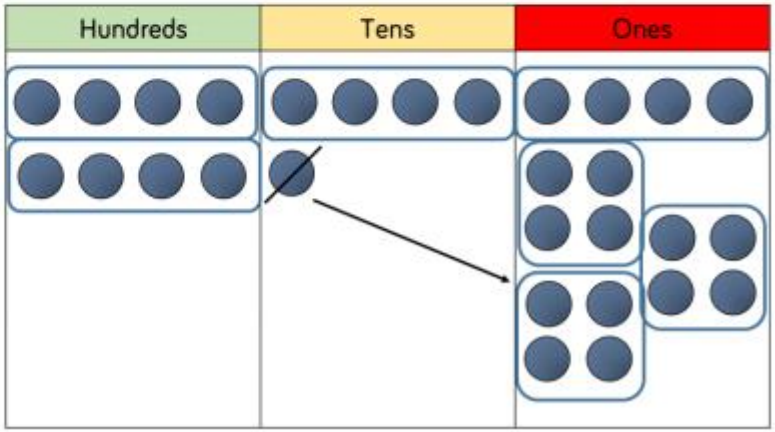
Stage 8: Year 5 objective

Skill: Divide 3-digits by 1-digit (grouping)

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



		2	1	4
	4	8	5	$^{1}6$



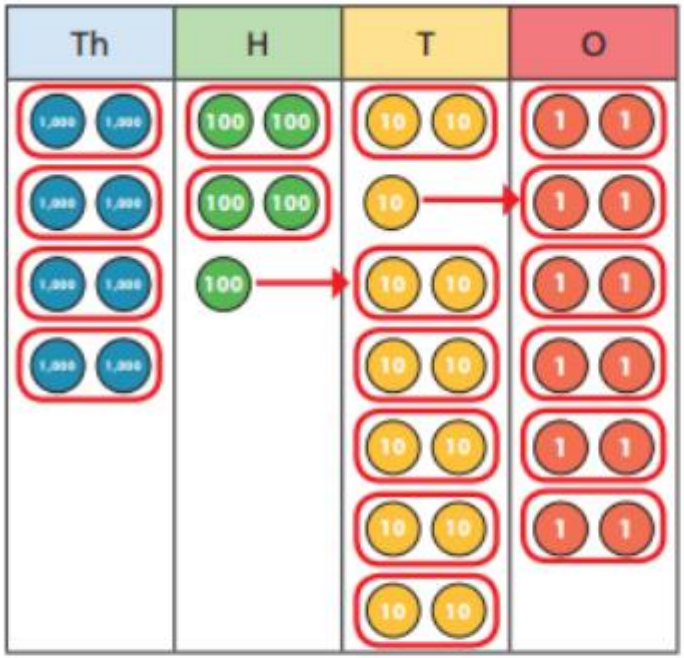
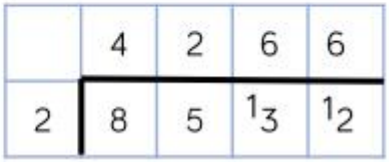
$856 \div 4 = 214$

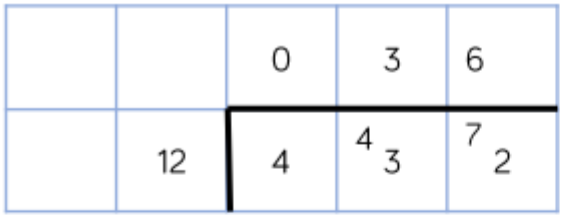
Children continue to use grouping to support their understanding of short division.

Place value counters or plain counters can be used on a grid to support their understanding.

Children can draw their own counters and group them to create a pictorial representation.

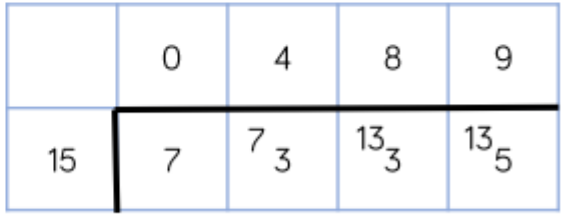
Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy

Stage 9: Year 5 objective	Skill: Divide 4-digits by 1-digit (grouping)
  $8{,}532 \div 2 = 4{,}266$	Place value counters or plan counters can be used on a place value grid. Children can create their own pictorial representation by drawing the counters onto a place value grid. Encourage children to move away from the concrete and pictorial representation when dividing numbers with multiple exchanges.
Stage 10: Year 6 objective	**Skill: Divide multi-digits by 2-digits (short division)**



$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$



15	30	45	60	75	90	105	120	135	150

At this stage, concrete and pictorial representations become less effective and written methods become the most accurate strategy.

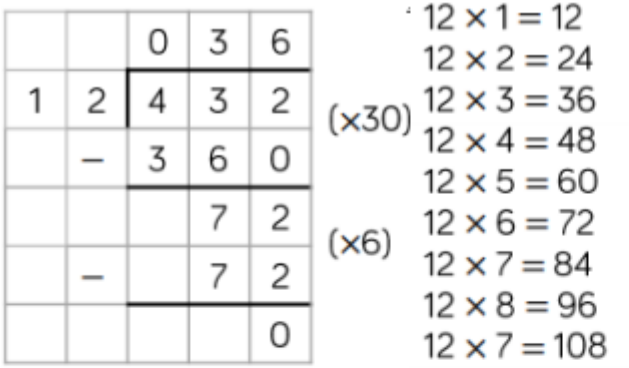
Children should write out multiples to support their calculations with larger remainders.

Children will also solve problems where the quotient can be rounded as appropriate.

Stage 11: Year 6 objective

Skill: Divide multi-digits by 2-digits (long division)

Written by Ben Paul, 2023
Reviewed by Teresa Ellington for Upwell Academy



		0	3	6
1	2	4	3	2
	–	3	6	0
			7	2
	–		7	2
				0

(×30)

(×6)

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children should write out multiples to support their calculations with larger remainders.

Children will also solve problems where the quotient can be rounded as appropriate.

Stage 12: Year 6 objective

Skill: Divide multi-digits by 2-digits (long division)

When a remainder is left at the end of a calculation, children can

$372 \div 15 = 24\ r12$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

$372 \div 15 = 24\frac{4}{5}$

either leave it as a remainder or convert it to a fraction.

This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.